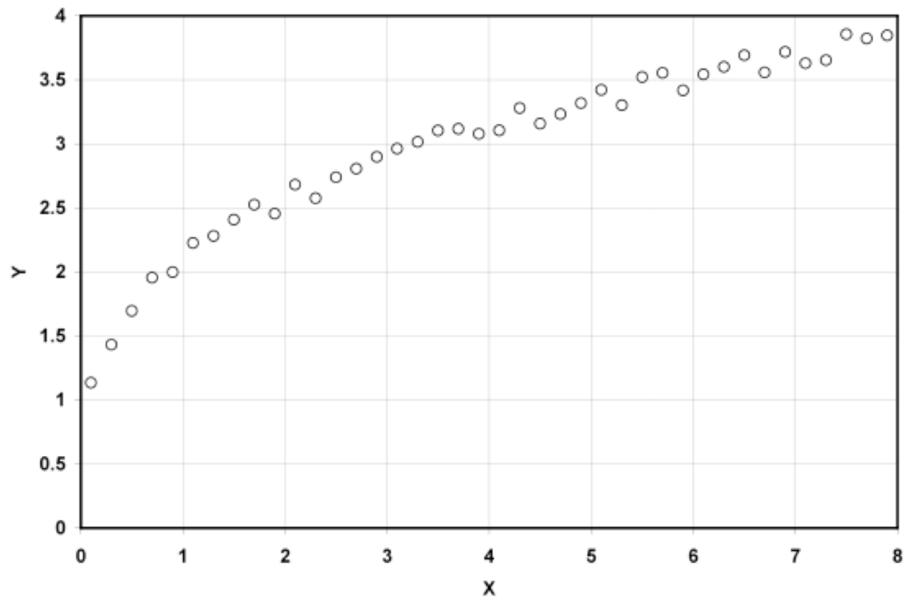
CE311K Spring 2009 **Example Exam 3** **April 30, 2009**

The exam is open book and open notes. You may use your course text book and your notes and handouts for this class.

ANSWERS at the bottom.

Problem 1. Consider the following graph of a set of data.



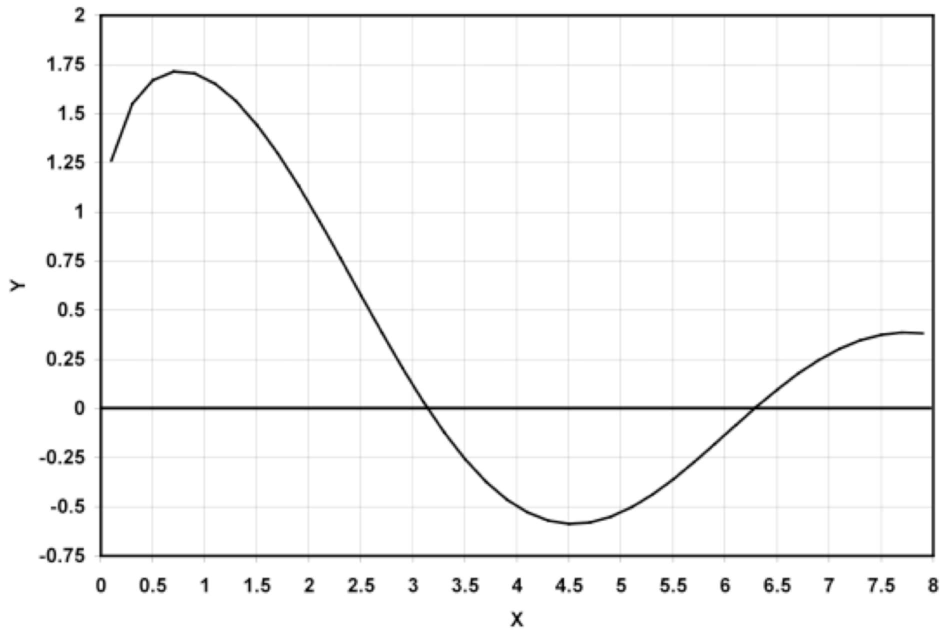
a. What kind of relationship (linear, nonlinear, power, exponential, etc.) do you hypothesize exists between x and y?

b. Write down the equation representing the relationship that you like for these data:

c. What are the unknown coefficients in your equation?

d. What equations would you need to solve to estimate the coefficients? Write them down.

Problem 2. Consider the following graph of a nonlinear function $y = f(x)$.



a. Write down the steps to carry out the first 2 iterations of the "***bisection***" method to solve for the smallest root of the function.

b. What is your estimate of the root after the second iteration?

c. What is the error in this estimate?

Problem 3. Given the following integral

$$I = \int_{0}^{3\pi/2} \sin(4x + 2)dx$$

a. Evaluate the above integral analytically

b. Using points running from $i = 0$ to $i = n$, write the equation for evaluating the integral numerically using Trapezoid rule, and

c. Using points running from $i = 0$ to $i = n$, write the equation for evaluating the integral numerically using Simpson's Rule.

ANSWERS

1.
   a. POWER
   b. $y = ax^b$
   c. $X = \ln(x) \qquad Y = \ln(y)$
      $a_0 = \ln(a) \qquad a_1 = b$
   d. a_0 and a_1
   e.

$$a_0 = \frac{\frac{1}{n}\sum_{i=1}^{n} Y_i \sum_{i=1}^{n} X_i^2 - \frac{1}{n}\sum_{i=1}^{n} X_i \sum_{i=1}^{n} X_i Y_i}{\sum_{i=1}^{n} X_i^2 - \frac{1}{n}\left[\sum_{i=1}^{n} X_i\right]^2}$$

$$a_1 = \frac{\sum_{i=1}^{n} X_i Y_i - \frac{1}{n}\sum_{i=1}^{n} X_i \sum_{i=1}^{n} Y_i}{\sum_{i=1}^{n} X_i^2 - \frac{1}{n}\left[\sum_{i=1}^{n} X_i\right]^2}$$

Then $a = e^{a_0} \qquad b = a_1$

2.
   a. Iteration 1)

```
a = 2, c = 4,
b = 0.5(a+c) = 0.5(2+4) = 3,
f(a)*f(b)= f(2)*f(3)>0;
a = b= 3
```

   Iteration 2)

```
a = 3, c = 4,
b = 0.5(a+c) = 0.5(3+4) = 3.5,
f(a)*f(b)= f(3)*f(3.5)>0;
c = b= 3.5
```

   b. Root = 3.5
   c. Less than ½ of the interval size = 1/2

3. a.

$$
\begin{aligned}
I &= \int_0^{3\pi/2} \sin(4x+2)dx \\
&= -\left.\frac{\cos(4x+2)}{4}\right|_0^{3\pi/2} \\
&= -\frac{\cos(6\pi+2) - \cos(2)}{4} \\
&= 0
\end{aligned}
$$

3. b.

$$I = \int_0^{3\pi/2} \sin(4x+2)dx \approx \frac{\Delta x}{2}[\sin(4*0+2) + 2*\sum_{i=1}^{n-1}\sin(4x_i+2) + \sin(4*3\pi/2+2)]$$

3. c.

$$I = \int_0^{3\pi/2} \sin(4x+2)dx \approx \frac{\Delta x}{3}[\sin(4*0+2) + 4*\sum_{\substack{i=1 \\ i\ odd}}^{n-1}\sin(4x_i+2) + 2*\sum_{\substack{i=1 \\ i\ even}}^{n-1}\sin(4x_i+2) + \sin(4*3\pi/2+2)]$$