Solving Nonlinear Equations

CE 311 K - Introduction to Computer Methods

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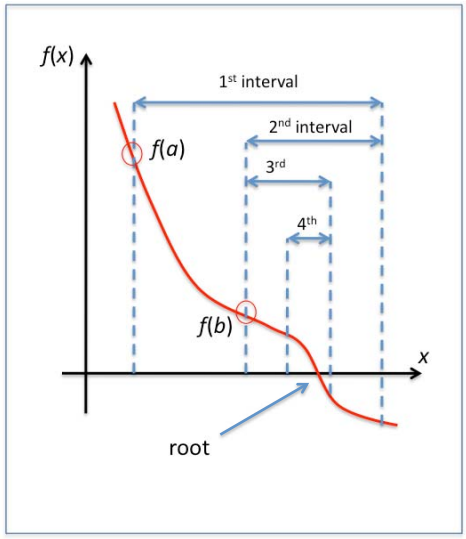
Solving Nonlinear Equations

- Three Methods
  - Fixed Point Iteration
  - Bisection
  - Newton

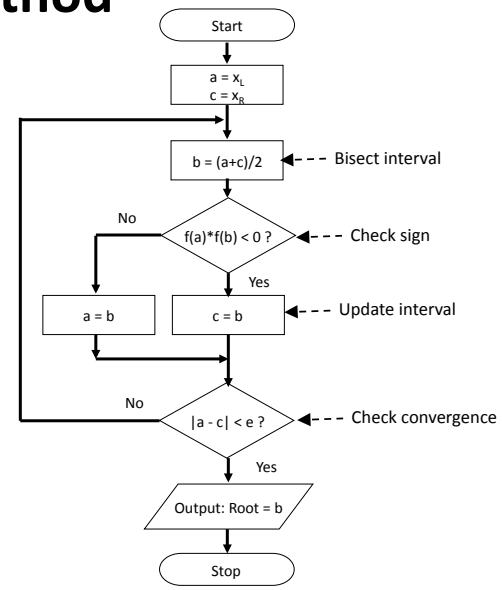
Bisection Method

- Bracket the root within smaller and smaller intervals

```
Do Until (|a – c| < tol)
      b=(a+c)/2
      If   f(a)*f(b) < 0  Then
            [a,b] contains the root
            Set c = b
      Else
            [b,c] contains the root
            Set a = b
      End If
Loop
```



Bisection Method

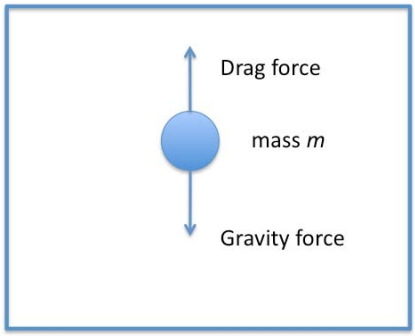


Example - Bisection

- Determine drag coefficient *x* for object of mass *m* with velocity *v* after free-falling for time *t*

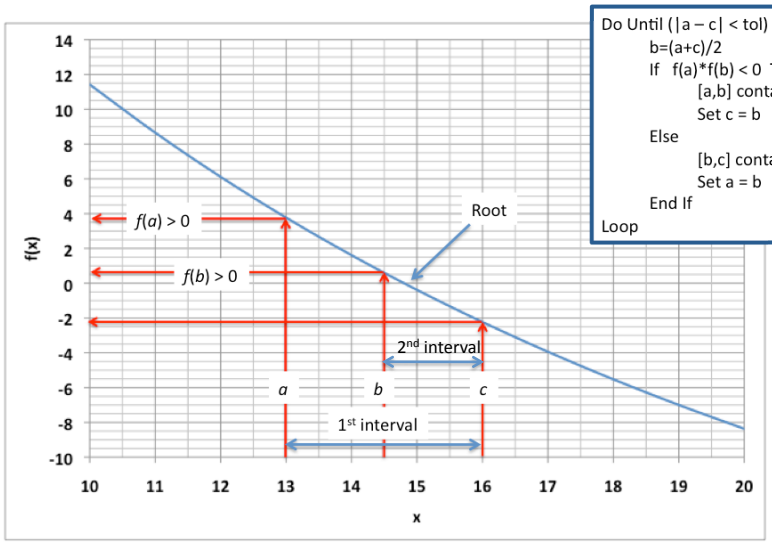
$$v(t) = \frac{mg}{x}\left[1 - e^{-(x/m)t}\right]$$

$$f(x) = \frac{mg}{x}\left[1 - e^{-(x/m)t}\right] - v = 0$$



m = 68.1 kg
v = 40 m/s
t = 10 s

Example - Bisection

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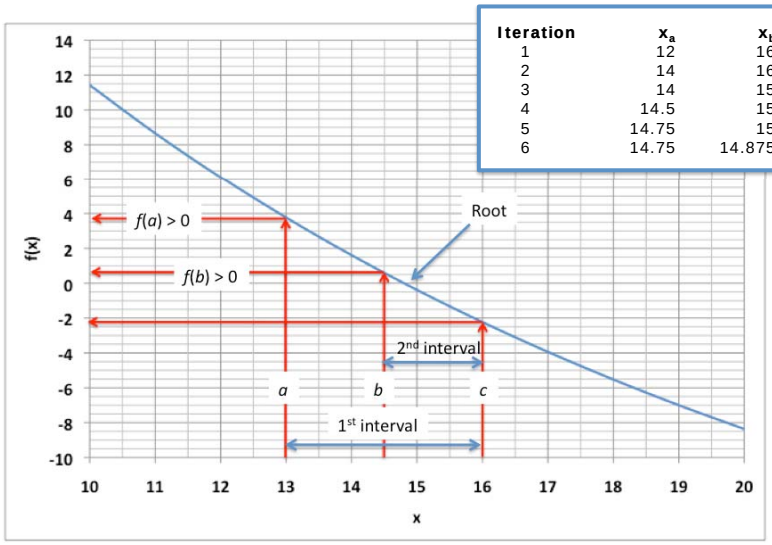
Example - Bisection



Iteration	x_a	x_b	x_c
1	12	16	14
2	14	16	15
3	14	15	14.5
4	14.5	15	14.75
5	14.75	15	14.875
6	14.75	14.875	14.8125

Excel Solver for Nonlinear Equations

- Mac: www.solver.com/mac/dwnmacsolver.htm

- **Colebrook's Formula**
  - Friction factor x for turbulent flow in a pipe (diameter D, roughness coefficient e, and Reynolds number Re)

$$\frac{1}{\sqrt{x}} = 1.14 - 2.0\log_{10}\left[\frac{\varepsilon}{D} + \frac{9.35}{\mathrm{Re}\sqrt{x}}\right]$$

  - Rearrange to give

$$f(x) = \frac{1}{\sqrt{x}} - 1.14 + 2.0\log_{10}\left[\frac{\varepsilon}{D} + \frac{9.35}{\mathrm{Re}\sqrt{x}}\right] = 0$$

Example - Solver

- Type the data into Cells B1 : B 4 as

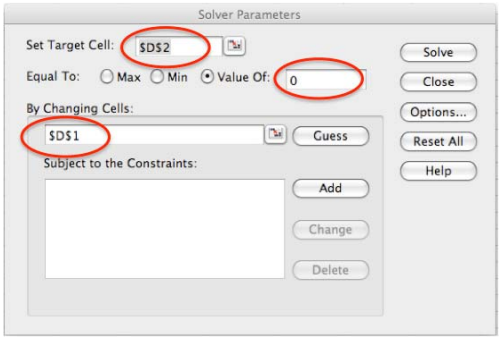
  D = 0.1 m, e = 0.0025 m, Re = 3 x 104
- Solver requires an initial value: Use x = 0.001
- In Cell C1 type " x = " and 0.001 in Cell D1
- In Cell C2 type " f(x) "and in Cell D2, type

=1/SQRT(D1)-1.14+2*LOG10(B2/B1+9.35/(B3*SQRT(D1)))

	A	B	C	D	E
1	D=	0.1	x=	0.001	
2	e=	0.0025	f(x)=	27.5673	
3	Re=	30000			
4	x0	0.001			
5					
6					

Example - Solver

- Activate "Solver"
- Set Target Cell
  - D2 "f(x)"
- Eqaul to Value of:
  - 0 "final value"
- By Changing Cells:
  - D1 "x"
- Solve
  - x = 0.0541
  - $f(x)$ = -4x10^{-07}



	A	B	C	D
1	D=	0.1	x=	0.05411
2	e=	0.0025	f(x)=	-4E-07
3	Re=	30000		
4	x0	0.001		