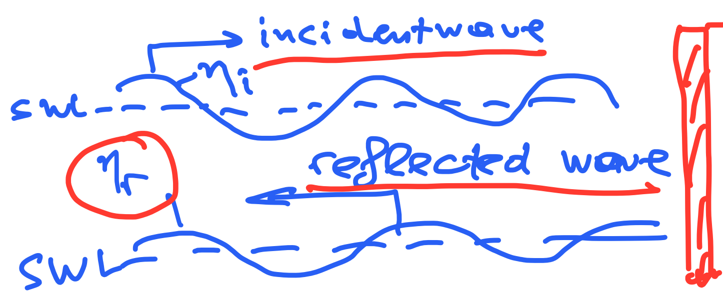


$\eta_i(x,t) = \frac{H}{2}\cos(kx-\omega t)$ *

$\eta_r(x,t) = \frac{H}{2}\cos(kx+\omega t)$ *

compound wave: $\eta_t(x,t) = \eta_i(x,t) + \eta_r(x,t) =$

$$= \frac{H}{2}\left[\cos(\underbrace{kx-\omega t}_{\alpha}) + \cos(\underbrace{kx+\omega t}_{\beta})\right] =$$

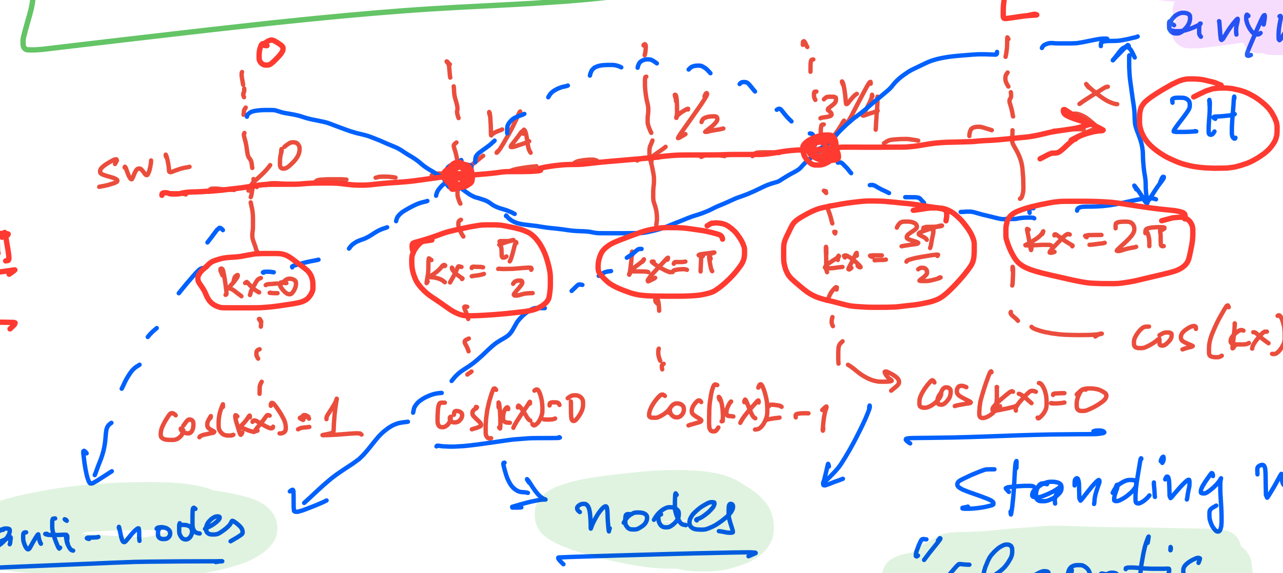
$$\cos(\alpha) + \cos(\beta) = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

$$= \frac{H}{\cancel{2}}\cancel{2}\cos\left[\frac{(kx-\cancel{\omega t})+(kx+\cancel{\omega t})}{2}\right]\cos\left[\frac{(\cancel{kx}-\omega t)-(\cancel{kx}+\omega t)}{2}\right] =$$

$$\Rightarrow \boxed{\eta_t(x,t) = \boxed{H\cos(kx)\cos(\omega t)}}$$

kx & ωt are NOT together anymore ☹

$k = \frac{2\pi}{L}$



Standing wave "clapotis"

Useful trigonometric IDs

(used in analysis of standing waves)

$$\sin(\alpha)+\sin(\beta)=2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

$$\sin(\alpha)-\sin(\beta)=2\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$$

$$\cos(\alpha)+\cos(\beta)=2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

$$\cos(\alpha)-\cos(\beta)=-2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$$

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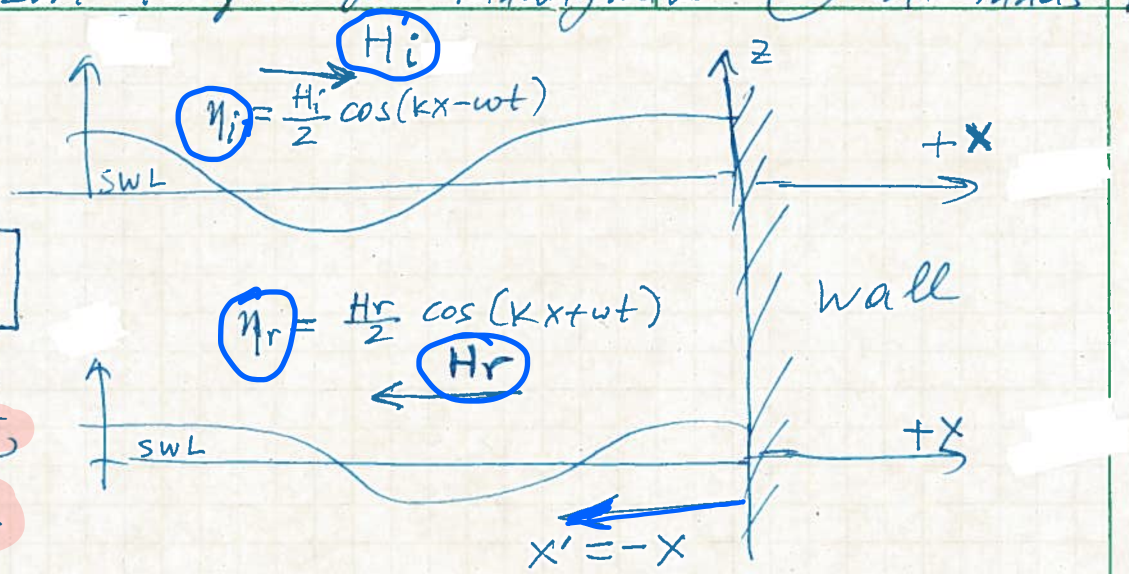
* In the next slides we examine the flowfield under a standing wave, in the case of deep water. The analysis can be easily extended in the case of transitional or shallow water.



Deep H_2O

perfect reflection

reflection coefficient,

$$\chi = \frac{H_r}{H_i} = 1$$

- Potential of incident wave: (deep H_2O)

$$\checkmark \; \cdot \Phi_i = \frac{a\omega}{k} e^{kz} \sin(kx-\omega t) \qquad a = \frac{H_i}{2}$$

- Potential of reflected wave:

We consider $H_r = H_i$ (perfect reflection)

$$\cdot \Phi_r = \frac{a\omega}{k} e^{kz} \sin(kx' - \omega t) \qquad a = \frac{H_r}{2}$$

$x' = -x$; $= -x$

Relative to x' the reflected wave goes to the "right"

$$\sin(k(-x)-\omega t)$$

$$\sin(-(kx+\omega t))$$

Finally $\checkmark$ $$\Phi_r = -\frac{a\omega}{k} e^{kz} \sin(kx+\omega t)$$

Due to linear theory (Φ_t = total potential = = potential of standing wave)

$$\Phi_t = \Phi_i + \Phi_r = \frac{a\omega}{k} e^{kz} \Big[\sin(\underbrace{kx-\omega t}_{\alpha}) - \sin(\underbrace{kx+\omega t}_{\beta})\Big] =$$

$$= \frac{2a\omega}{k} e^{kz} \left[\cos\Big(\underbrace{\frac{\alpha+\beta}{2}}_{\frac{(kx-\cancel{\omega t})+(kx+\cancel{\omega t})}{2}}\Big) \sin\Big(\underbrace{\frac{\alpha-\beta}{2}}_{\frac{(\cancel{kx}-\omega t)-(\cancel{kx}+\omega t)}{2}}\Big)\right]$$

Finally:

$$\Phi_t = -\frac{2aw}{k} e^{kz} \cdot \cos(kx) \cdot \sin(\omega t)$$

total potential for the standing wave

velocities of particles under standing wave

$$u_t = \frac{\partial \phi_t}{\partial x} = \boxed{2aw\, e^{kz} \sin(kx) \sin(\omega t)}$$

$$w_t = \frac{\partial \phi_t}{\partial z} = \boxed{-2aw e^{kz} \cos(kx) \sin(\omega t)}$$

accelerations of particles under standing wave

$$a_{x,t} = \frac{\partial u_t}{\partial t} = 2a\omega^2 e^{kz} \sin(kx) \cos(\omega t)$$

$$a_{z,t} = \frac{\partial w_t}{\partial t} = -2a\omega^2 e^{kz} \cos(kx) \sin(\omega t)$$

For the wave pressures:

(For gage pressure: $p_t^{gage} = \rho g \eta_t e^{kz} - \rho g z$)

wave pressure under standing wave

$$\boxed{p_i^{wave} = \rho g \eta_i e^{kz}} \quad ; \quad \boxed{p_r^{wave} = \rho g \eta_r e^{kz}}$$

$$p_t^{wave} = p_i^{wave} + p_r^{wave} = \rho g (\eta_i + \eta_r) e^{kz} = \rho g \eta_t e^{kz}$$

same as what we know!

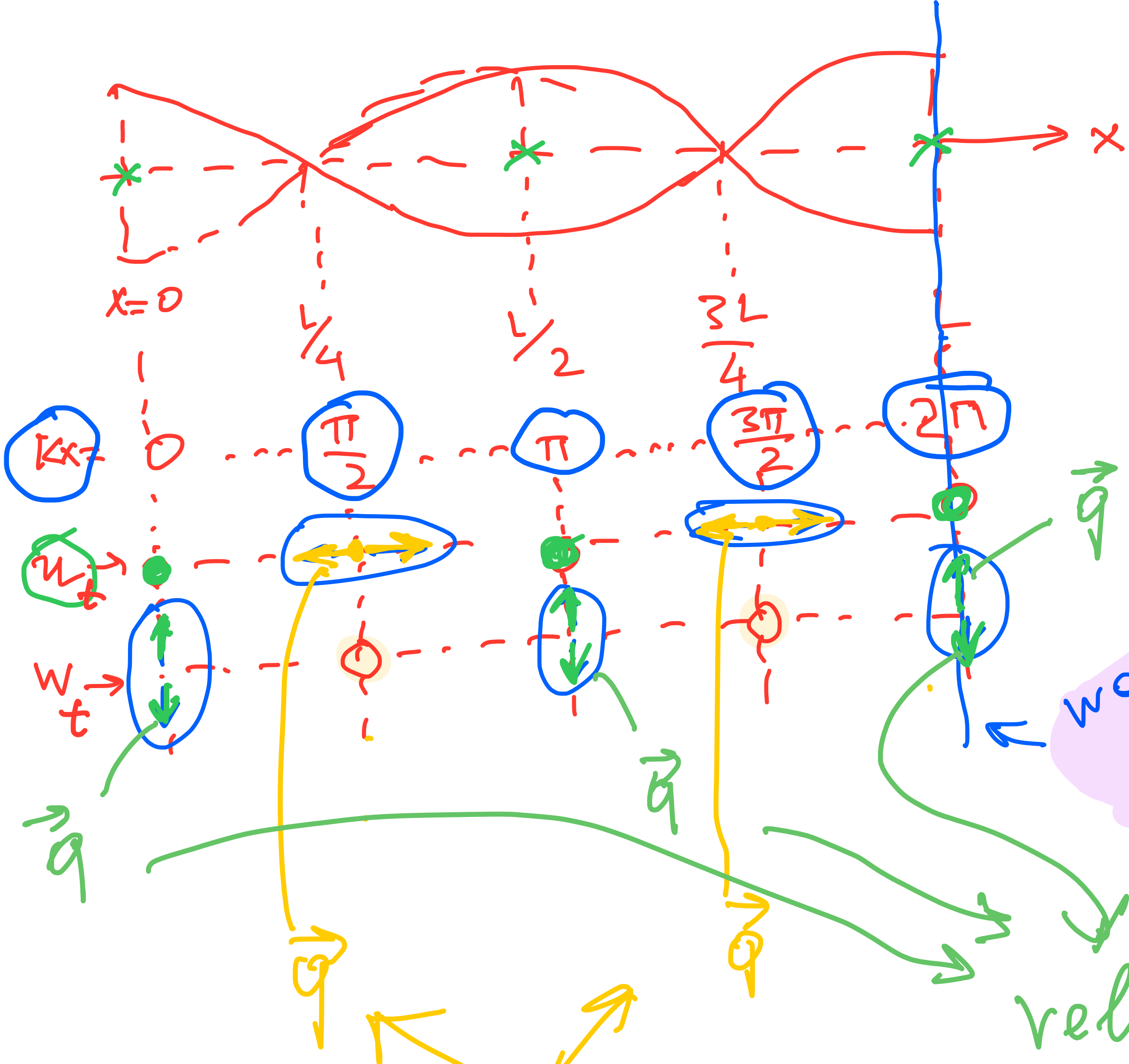
$$\eta_i = a\cos(kx - \omega t)$$

$$\eta_r = a\cos(kx' - \omega t) = a\cos(-kx - \omega t) = a\cos(kx + \omega t)$$

$$\boxed{\eta_t} = \eta_i + \eta_r = a\left[\cos(\underbrace{kx - \omega t}_{\alpha}) + \cos(\underbrace{kx + \omega t}_{\beta})\right] =$$

$$= 2a\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right) = \boxed{2a\cos(kx)\cos(\omega t)} = \boxed{H\cos(kx)\cos(\omega t)}$$

$$u_t = 2a\omega e^{kz}\sin(kx)\sin(\omega t)$$
$$w_t = -2a\omega e^{kz}\cos(kx)\sin(\omega t)$$



wall has to be an anti-node

velocity vectors at anti-nodes

velocity vectors at nodes

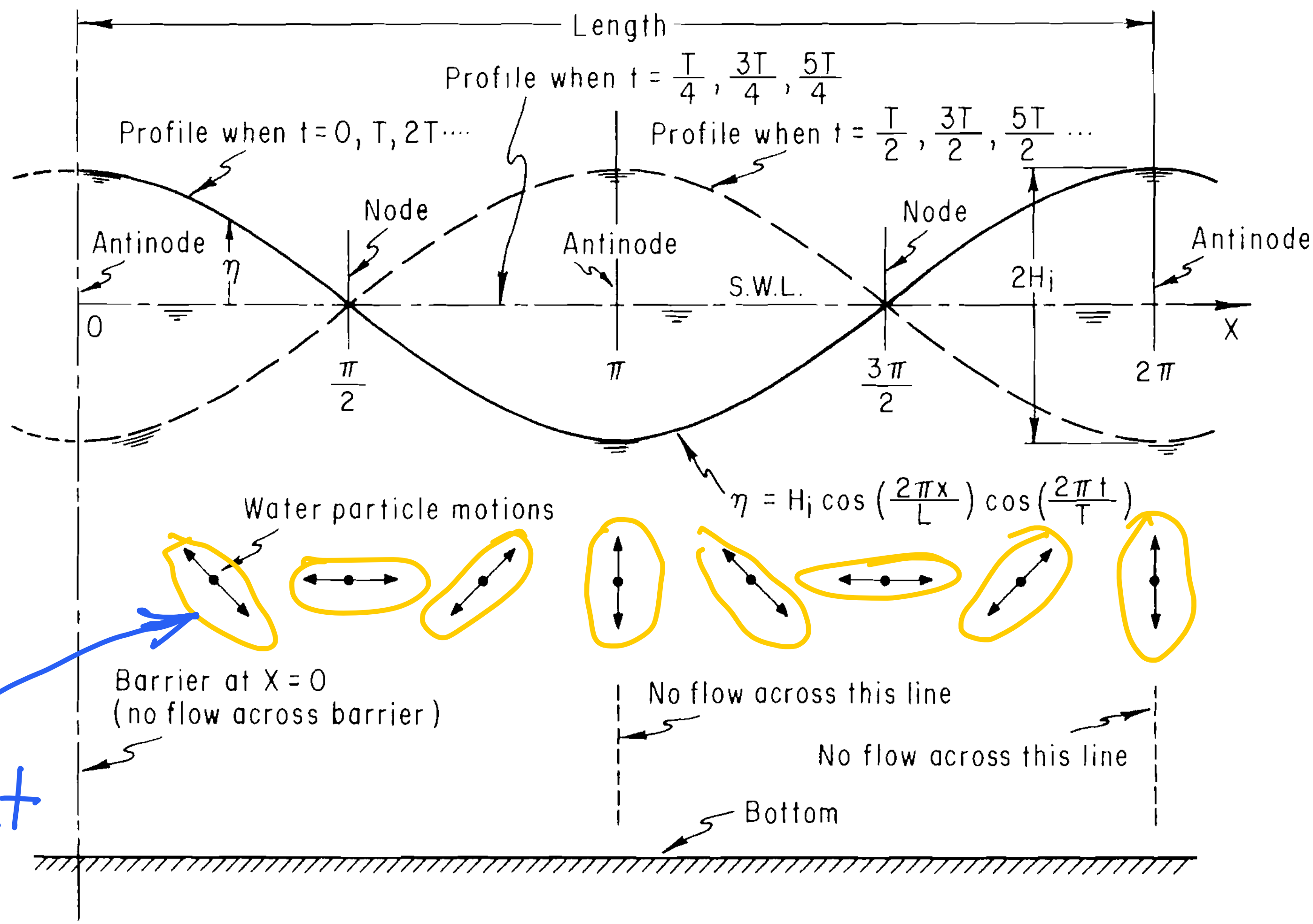


Figure 2-63. Standing wave (clapotis) system, perfect reflection from a vertical barrier, linear theory.

see next slides for an experimental verification!

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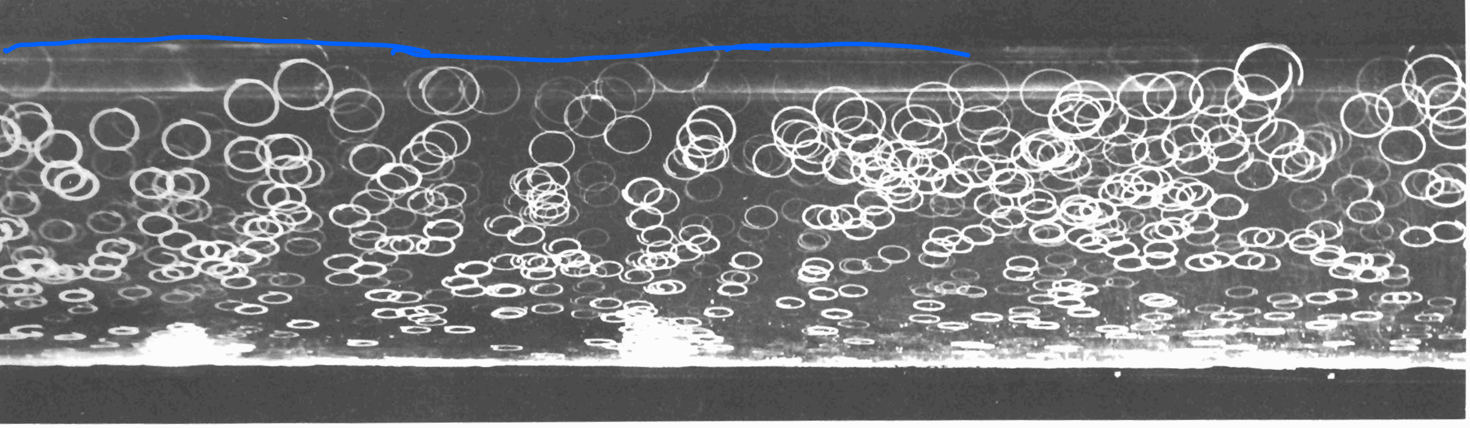
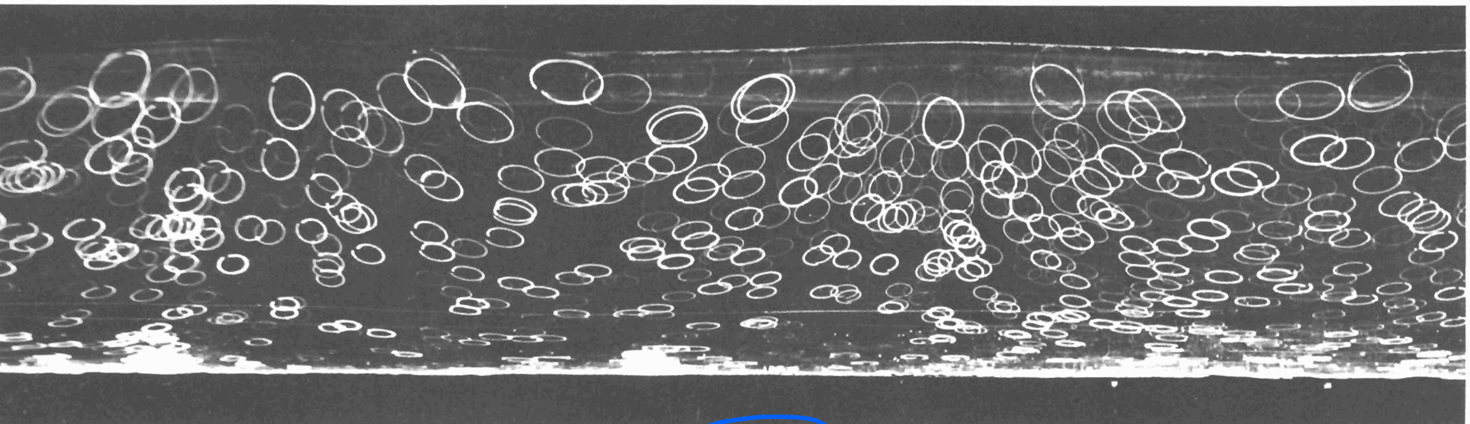
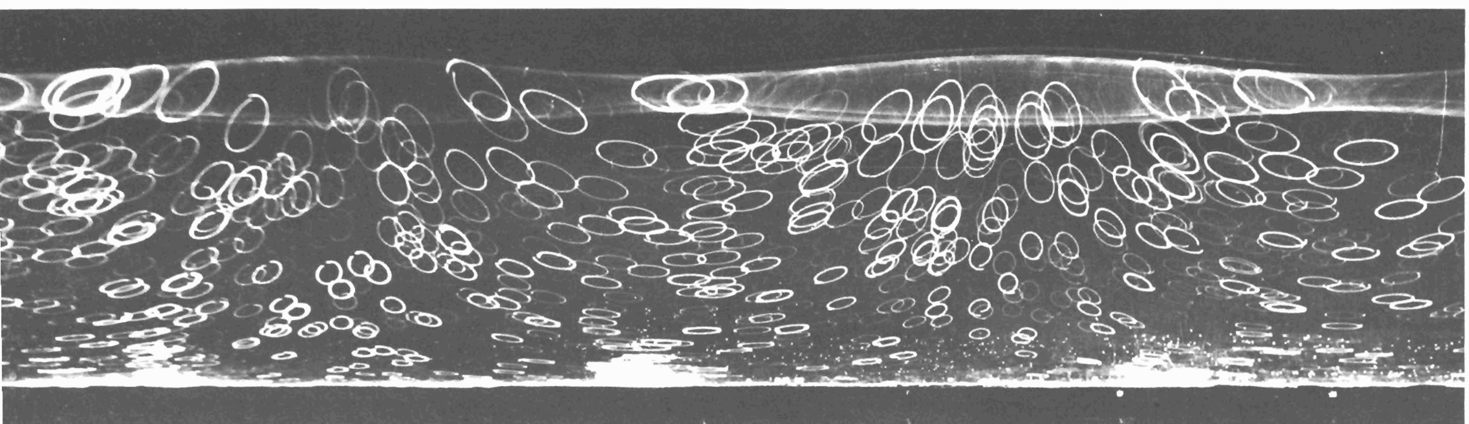


No reflection: pure progressive waves

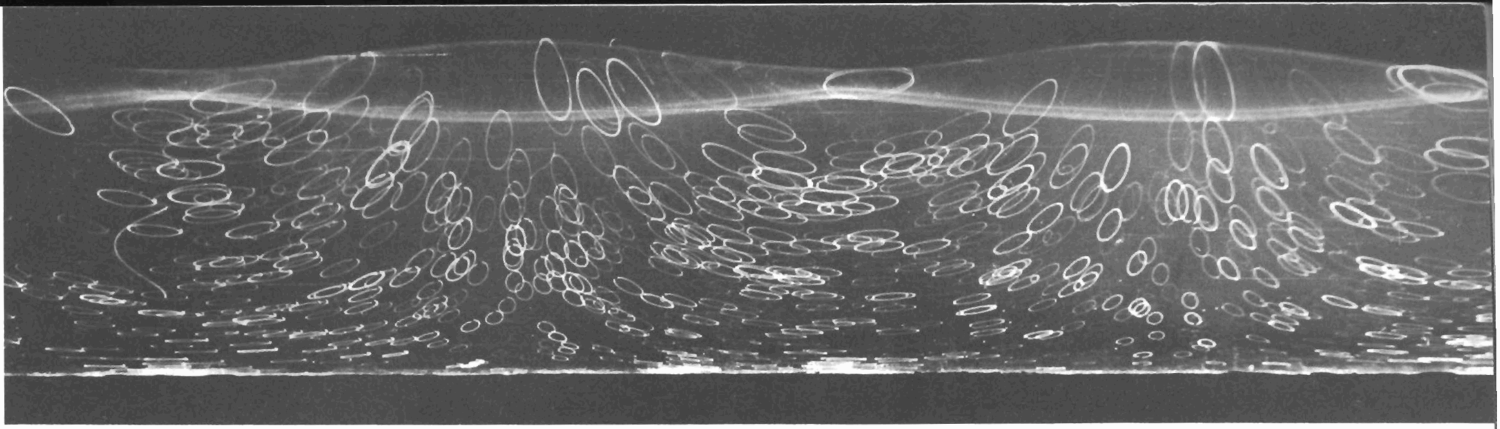


24% reflection

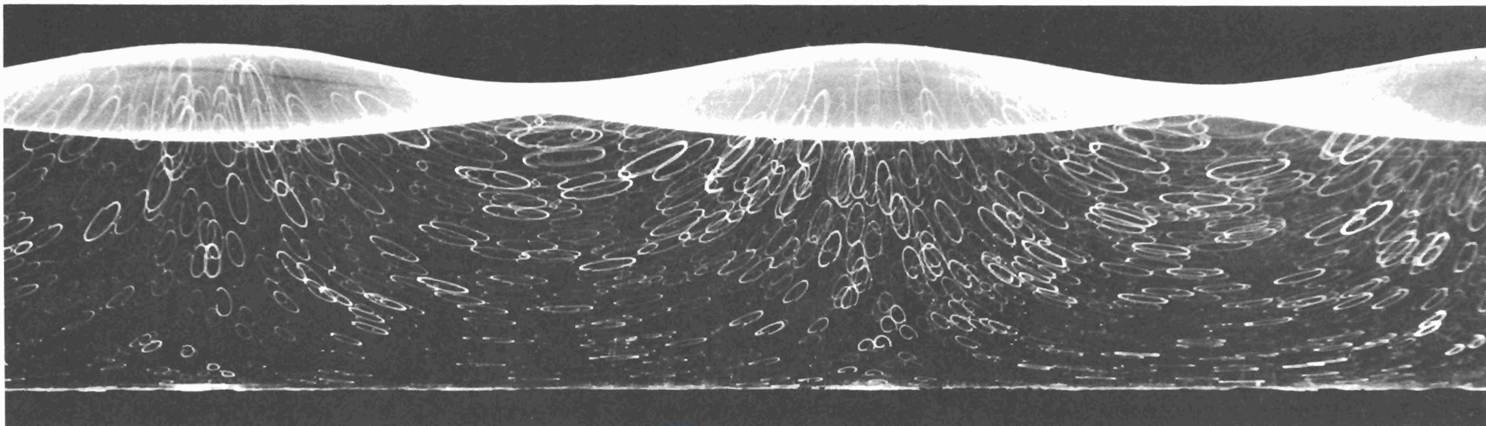


38% reflection

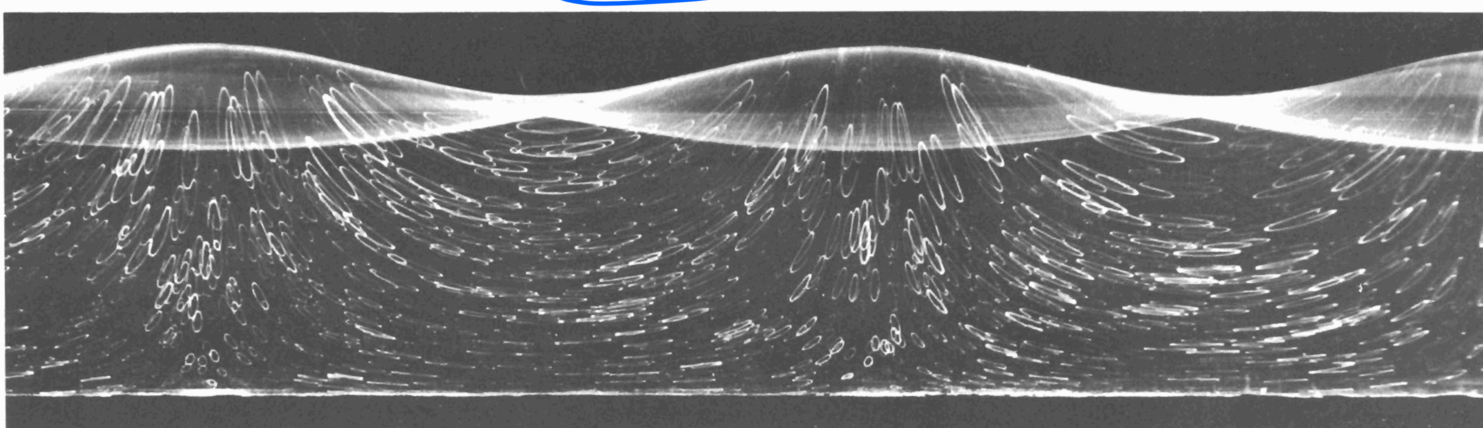
191. Particle trajectories in plane periodic water waves. Two wave trains of the same frequency traveling in opposite directions are produced by a progressive wave coming from the left that is reflected by a partially absorbent barrier. The top photograph shows the pure progressive wave with no reflection. Its amplitude is four per cent of the wavelength, and the water depth is 22 per cent. White particles suspended in the water are photographed during one period. Their trajectories are practically ellipses traversed clockwise, circular at the free surface and flattened toward the bottom. Some open loops indicate a slow drift to the right near the surface and left near the bottom. As the reflection is increased, the orbits become increasingly flattened and inclined. Complete reflection gives a pure standing wave in the last photograph, where the trajectories are streamlines. There the upper and lower envelopes of the water surface show that the vertical motion does not vanish at the nodes. *Wallet & Ruellan 1950, courtesy of M. C. Vasseur*



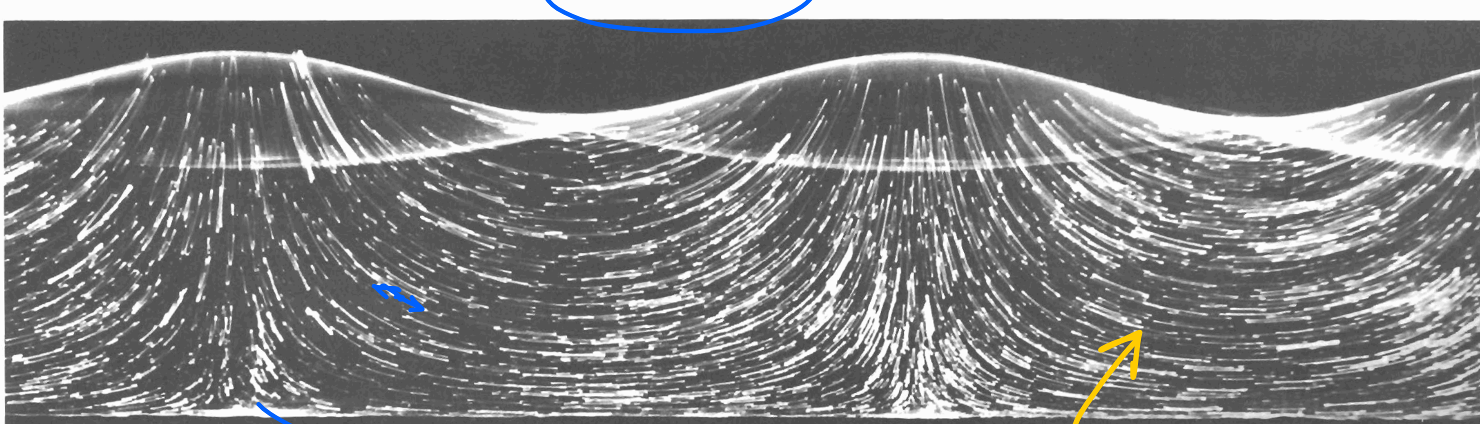
53% reflection



71% reflection



85% reflection

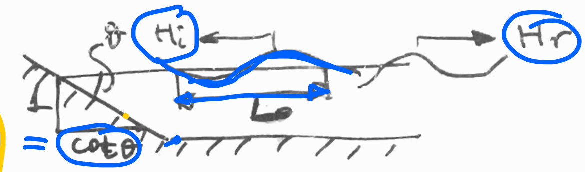


100% reflection pure standing waves

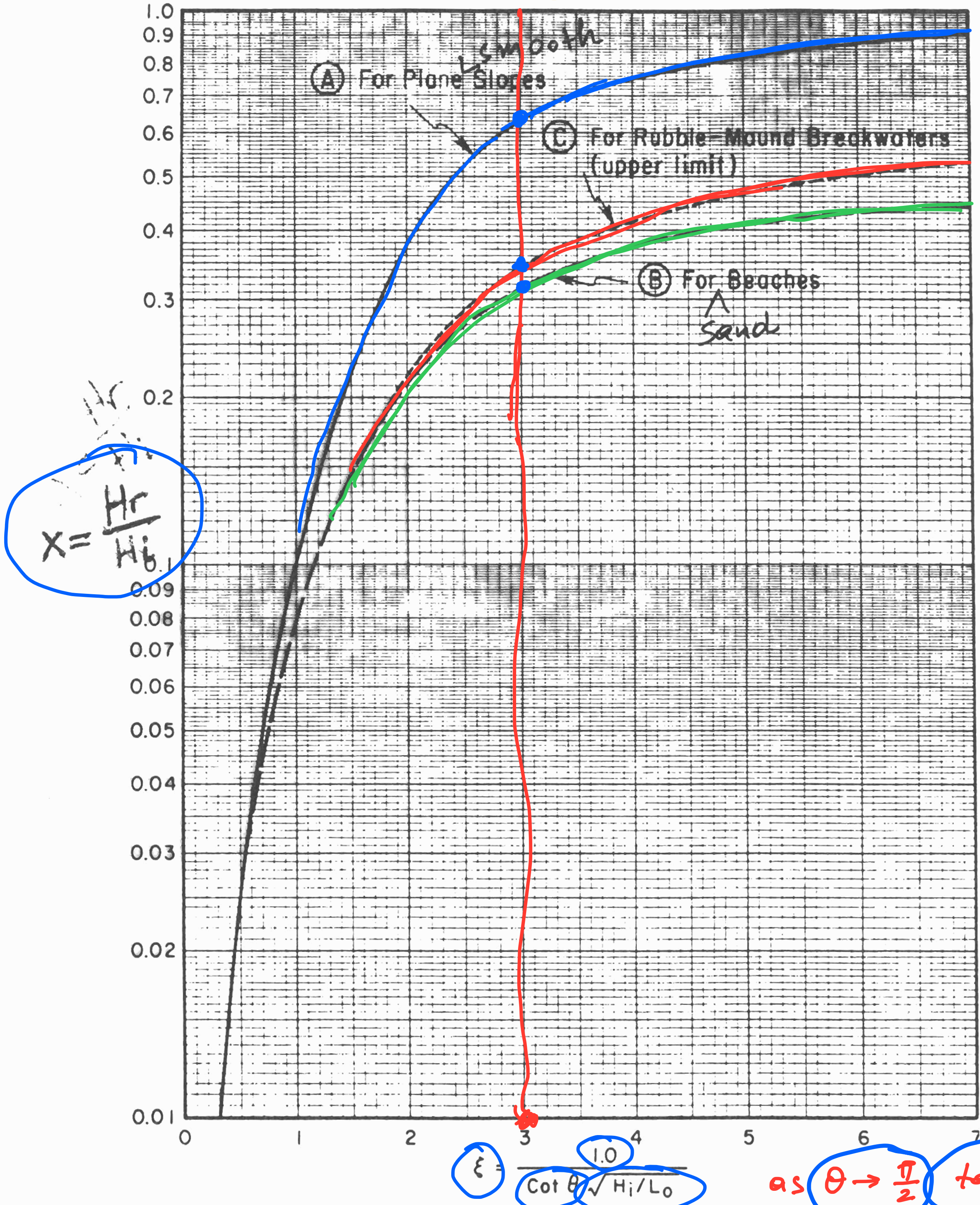
$\chi = 1$

particle trajectories are straight segments for $\chi = 1$

steeper waves are dissipated more!



$\frac{1}{\tan\theta} = \cot\theta$



as $\theta \to \frac{\pi}{2}$ $\tan\frac{\pi}{2} \to \infty$ $\xi \to \infty$

Figure 2-65. Wave reflection coefficients for slopes, beaches, and rubble-mound breakwaters as a function of the surf similarity parameter ξ.

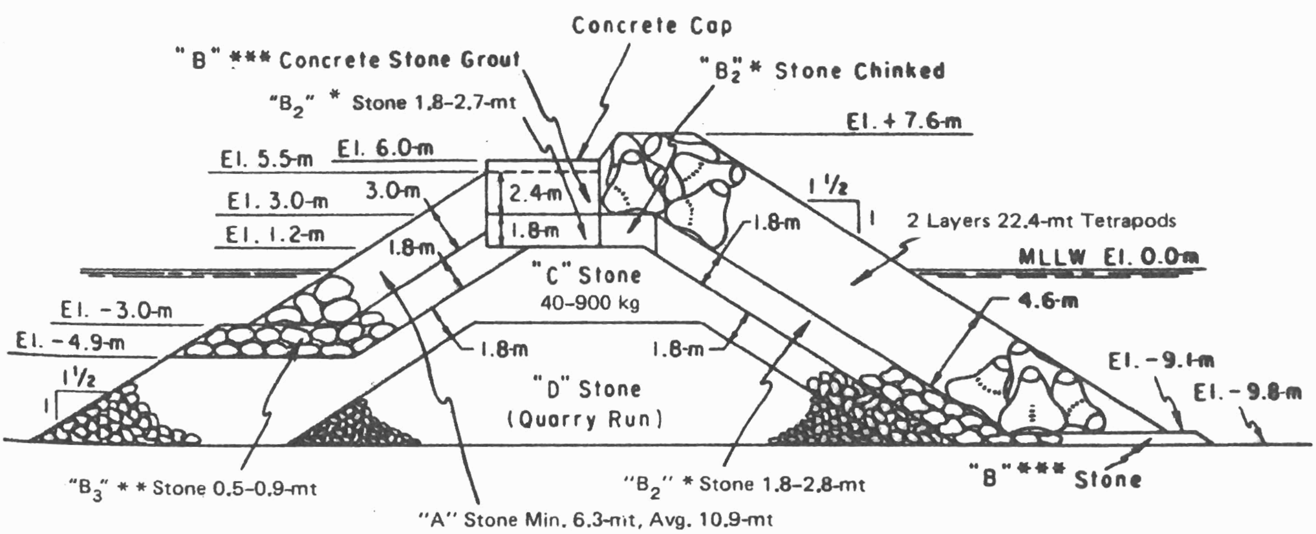
$L_0 = \frac{gT^2}{2\pi}$

as T or L_o ↑↑ $\xi \to \infty$ ⇒ $X \to 1$

Tsunamis do not dissipate easily and can travel long distances

Tetrapod rubble-mound breakwater

Cresent City, California (Apr. 1964)



* "B_2" - 0.9-mt Variation to 6.3-mt Max.
** "B_3" - 0.5-to 0.9-mt Min.; 6.3-mt Max. as Available
*** "B" - 0.9-to 6.3-mt or to Suit Depth Conditions at Seaward Toe

Figure 6-63. Tetrapod and rubble-mound breakwater.

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