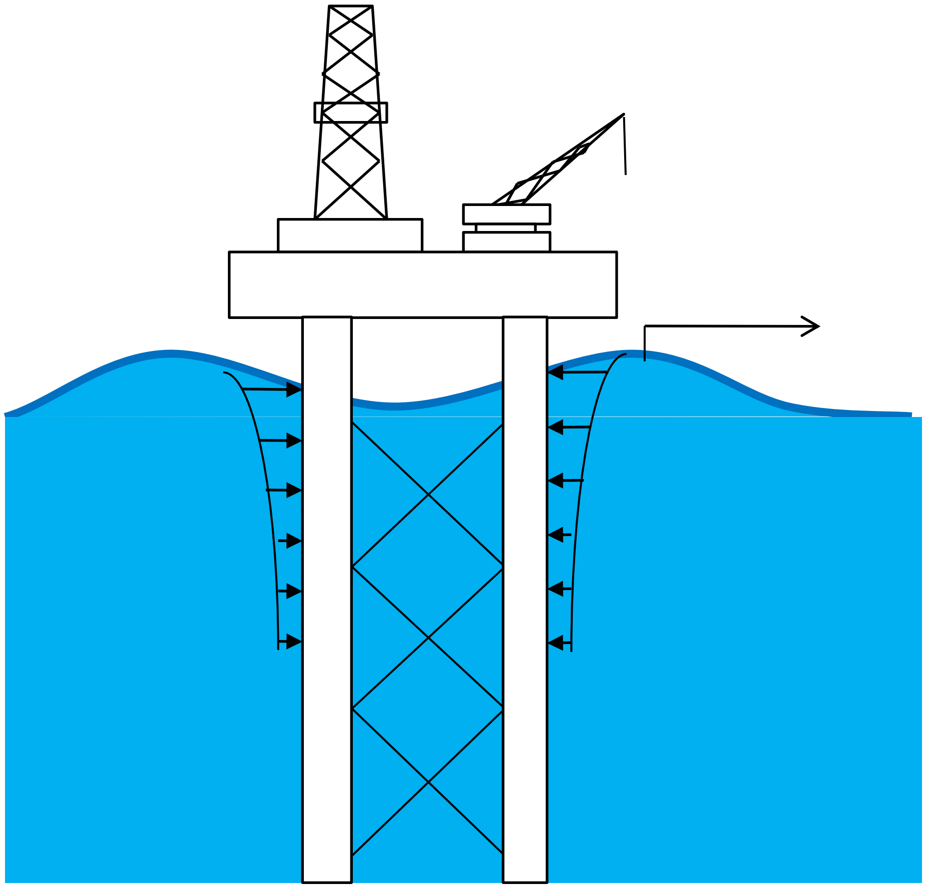
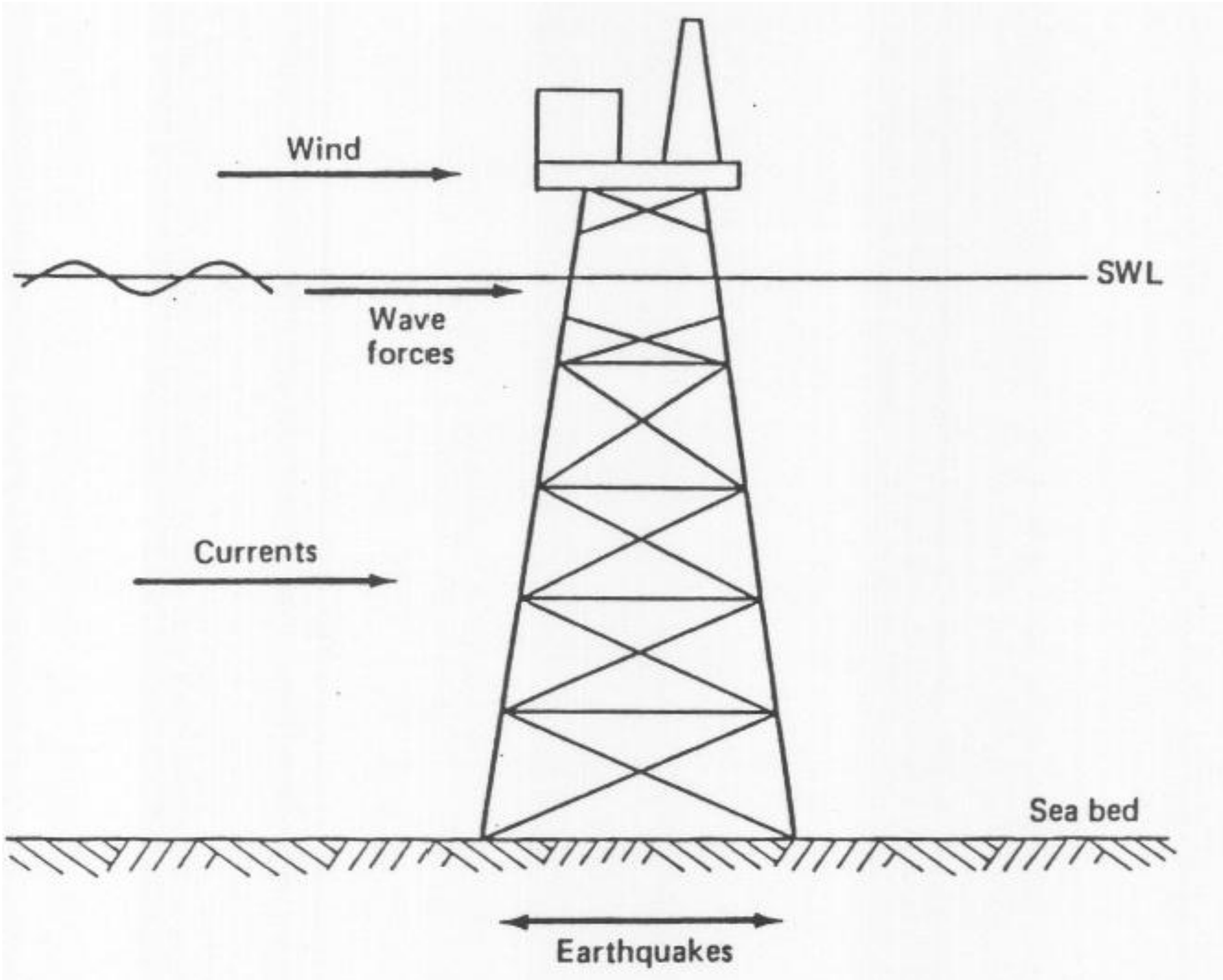
Wave, current, and wind forces



An offshore structure is designed to withstand the 100-year storm (wave/current/wind). A mono-chromatic wave of height H_{max} is assumed.


Wind
SWL
Wave
forces
Currents
Sea bed
Earthquakes

OFFSHORE PLATFORMS are comprised of many cylindrical or prismatic components (structural elements, floatation parts, risers, tendons, mooring lines, etc)

Spar

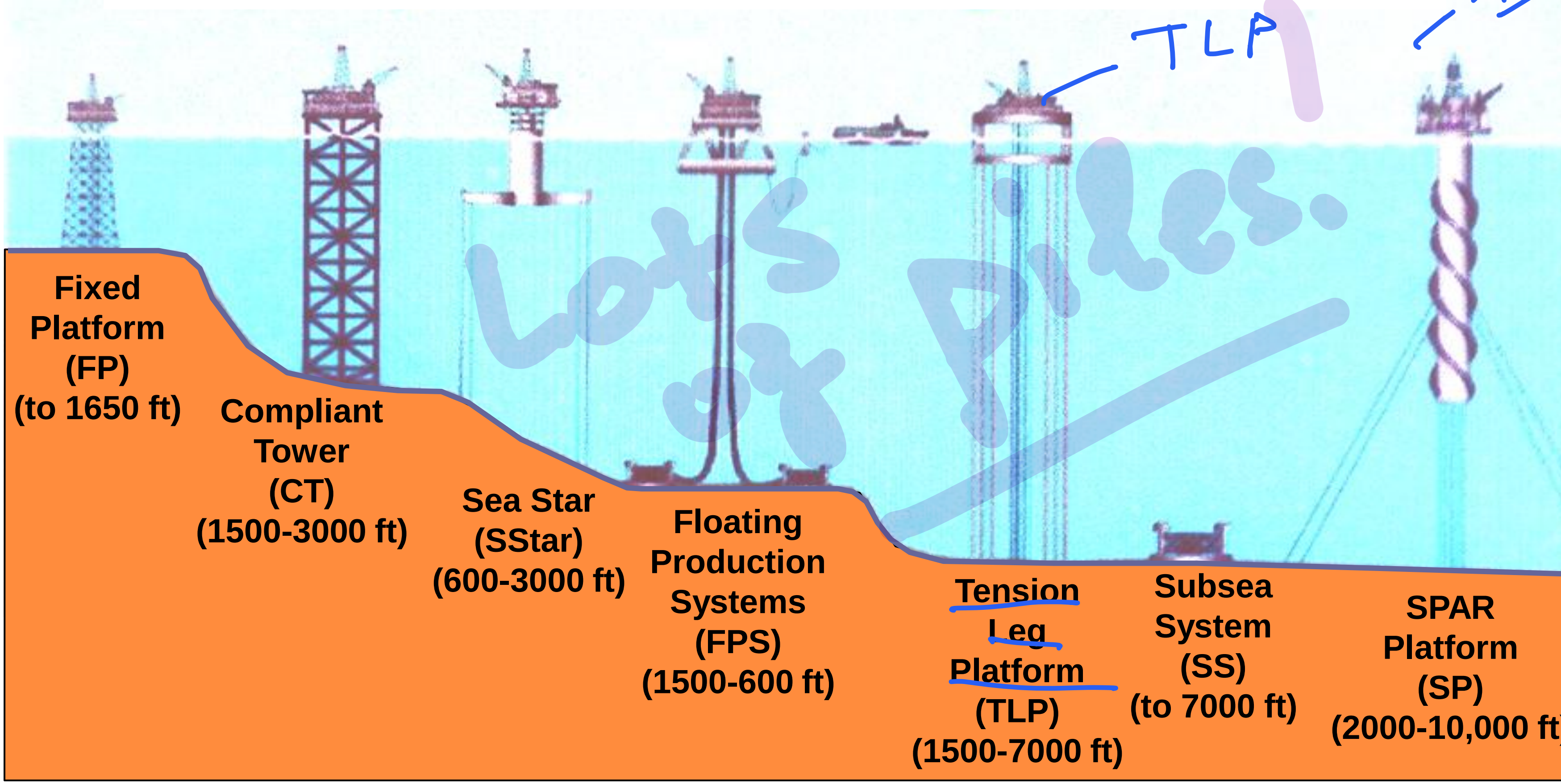
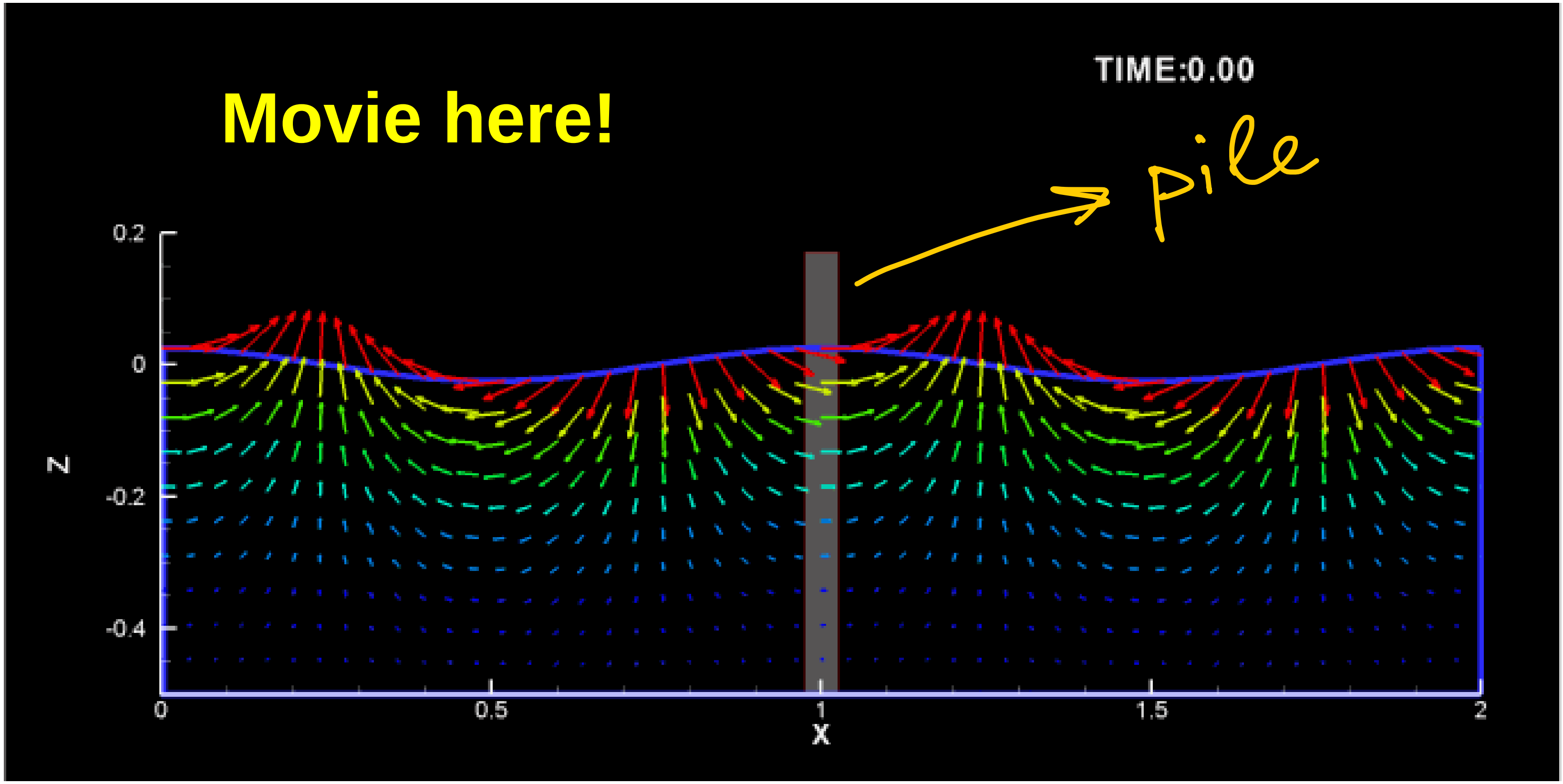
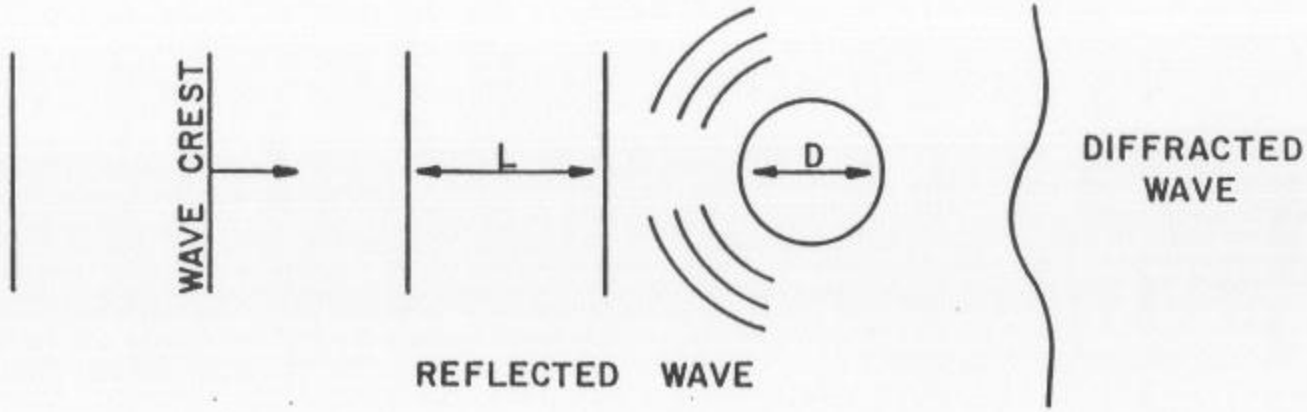


Figure from BOEMRE, U.S. Department of the Interior

What inflow velocity would a pile be subjected to?

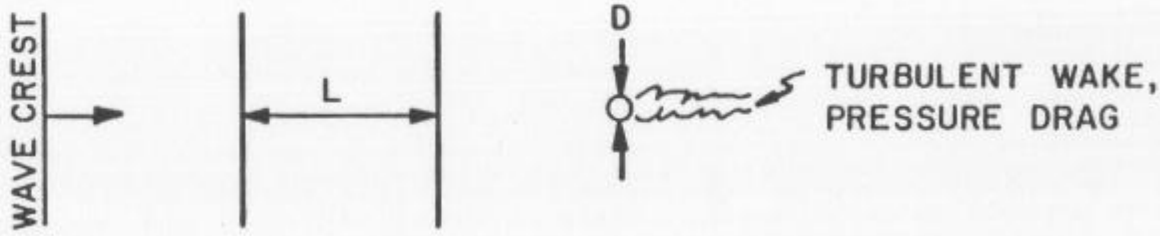


WAVE FORCE LIMITING CASE



WAVES MODIFIED BY OBJECT, $D > \frac{L}{5}$

WAVE FORCE LIMITING CASE



WAVE DOES NOT "FEEL" OBJECT, $D<\frac{L}{6}$

Steps to take:

- Study steady flow around 2-D cylinder (circle) subject to **steady inflow**

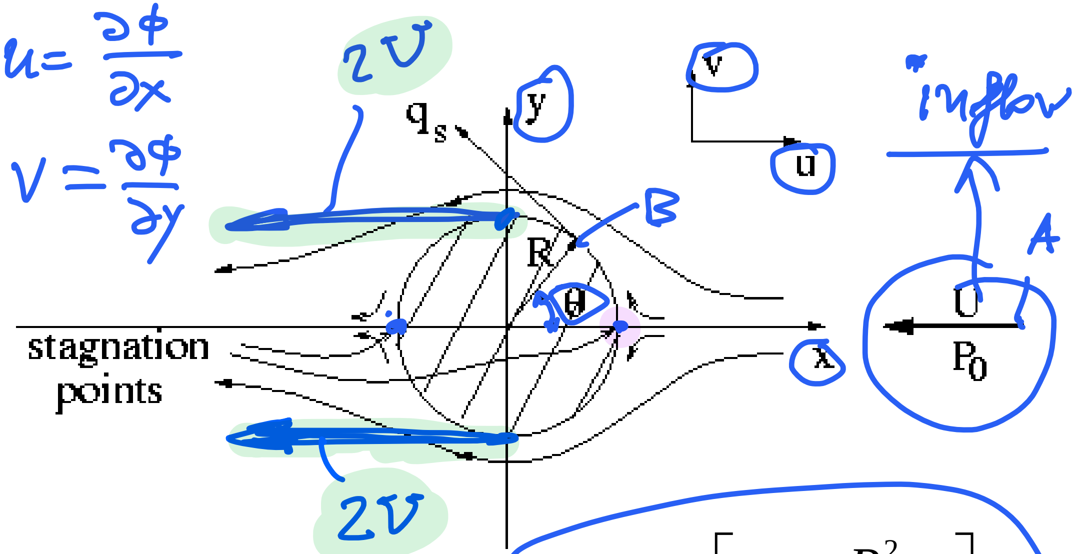
- Study unsteady flow around 2-D cylinder (circle) subject to **accelerating inflow**

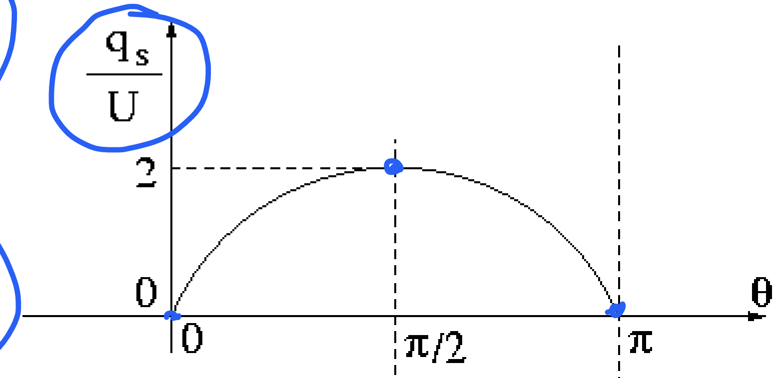
- Apply the study and the formulas developed in the previous steps, on "slices" of the **3-D cylinder, subject to wave and current,** and integrate along its length to determine total forces and moments

Inviscid Flow Around a Circle

$u = \frac{\partial \phi}{\partial x}$

$v = \frac{\partial \phi}{\partial y}$





Velocity potential:

$$\Phi = -Ux\left[1 + \frac{R^2}{x^2 + y^2}\right]$$

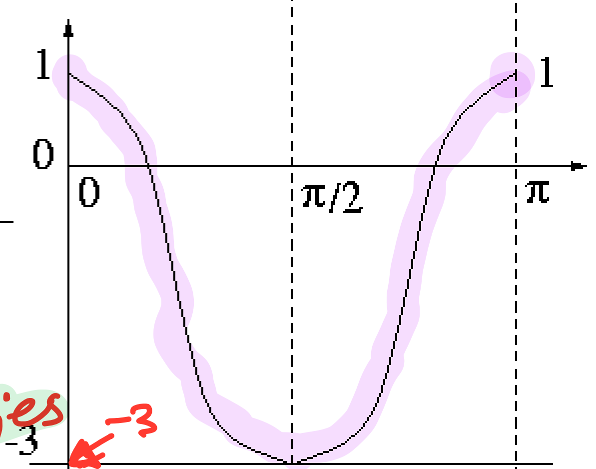
Surface velocity

$$q_s = \sqrt{u^2 + v^2} = 2U \sin \vartheta$$

Surface pressure coefficient

$$C_P = \frac{P_S - P_0}{\frac{1}{2}\rho U^2} = 1 - \left(\frac{q_S}{U}\right)^2$$

$$C_p = \frac{P_s - P_0}{\frac{1}{2}\rho U^2}$$



Also true for general shape 3-D bodies

D’Alembert “paradox”: The force on a body subject to inviscid steady flow is equal to zero!

Bernoulli between (A) far upstream and (B) on the surface of the circle:

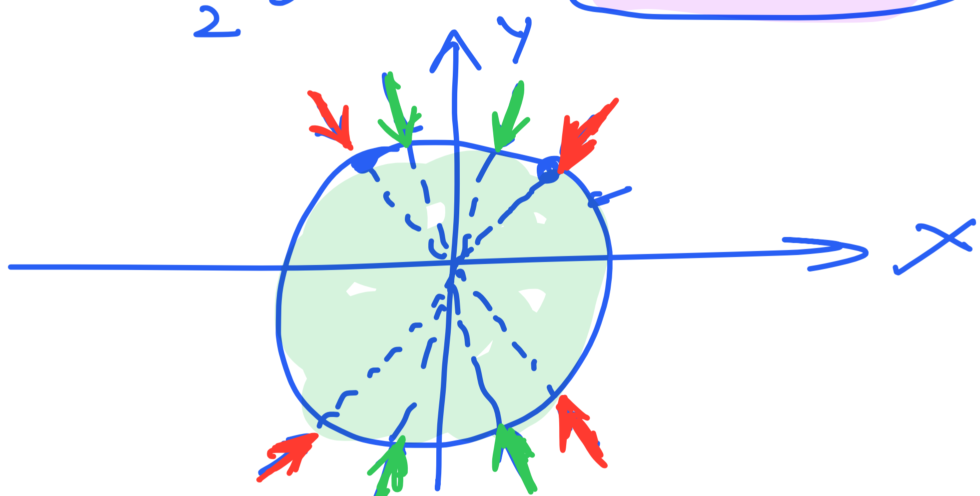
$$p_A + \frac{\rho}{2} U_A^2 = p_B + \frac{\rho}{2} v_B^2$$

we assume cylinder is horizontal $\Delta z = 0$

$$p_0 + \frac{\rho}{2} U^2 = p_B + \frac{\rho}{2} q_s^2$$

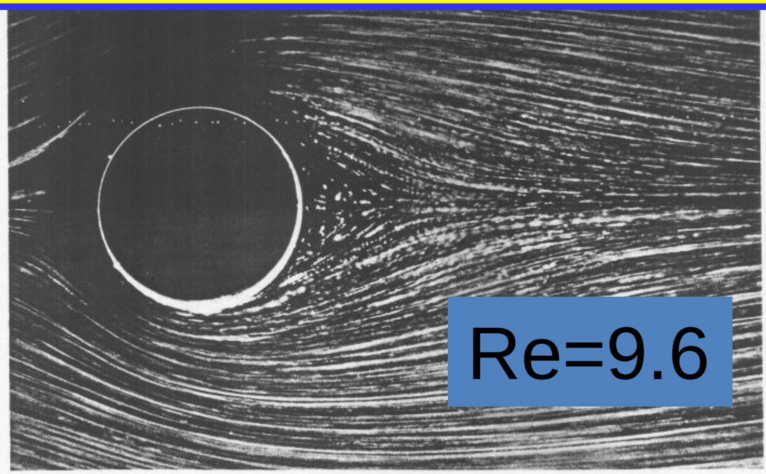
$$\boxed{C_p} = \text{pressure coefficient} = \frac{p_B - p_0}{\frac{\rho}{2} U^2} =$$

$$= \frac{\frac{\rho}{2} U^2 - \frac{\rho}{2} q_s^2}{\frac{\rho}{2} U^2} = \boxed{1 - \left(\frac{q_s}{U}\right)^2} \longrightarrow \text{(unitless)}$$

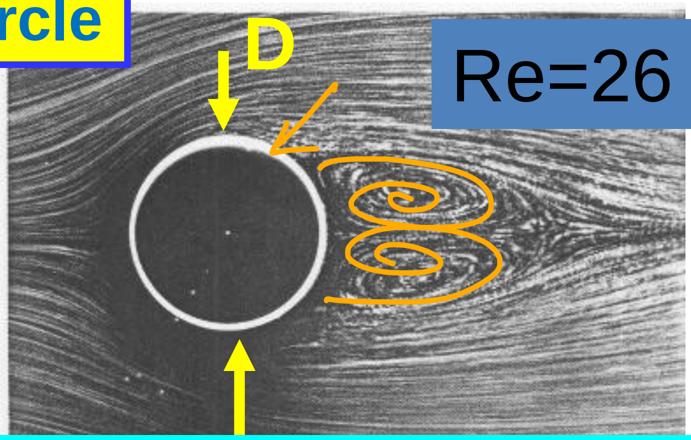


symmetry of pressures w.r.t. to x or y results into ZERO FORCE

Effect of viscosity on flow around circle

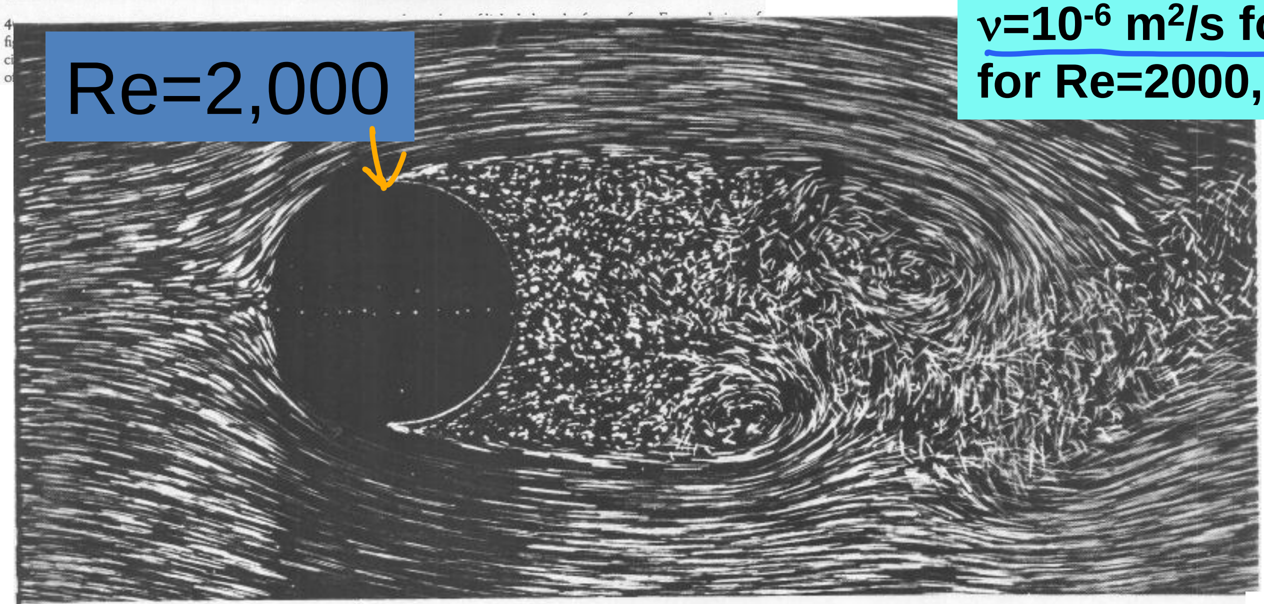


U



$\nu = \mu / \rho =$ kinematic viscosity

ν=10^{-6} m^2/s for H_2O at 20^o C. If D=20cm for Re=2000, U should be 0.01 m/s



47. Circular cylinder at R=2000. At this Reynolds number one may properly speak of a boundary layer. It is laminar over the front, separates, and breaks up into a turbulent wake. The separation points, moving forward as the Reynolds number is increased, have now attained their upstream limit, ahead of maximum thickness. Visualization is by air bubbles in water. *ONERA photograph, Werlé & Gallon 1972*

Reynolds Number

$$\mathrm{Re} = \frac{UD}{\nu}$$

Photos from Album of Fluid Motion of M. Vandyke

Air

Physical properties of air and water ~~circle~~

H_2O

Table A.3 MECHANICAL PROPERTIES OF AIR AT STANDARD ATMOSPHERIC PRESSURE

Temperature	Density ρ	Specific Weight	Dynamic Viscosity μ	Kinematic Viscosity ν
	kg/m³	N/m³	N · s/m²	m²/s
−20°C	1.40	13.70	1.61×10^{-5}	1.16×10^{-5}
−10°C	1.34	13.20	1.67×10^{-5}	1.24×10^{-5}
0°C	1.29	12.70	1.72×10^{-5}	1.33×10^{-5}
10°C	1.25	12.20	1.76×10^{-5}	1.41×10^{-5}
20°C	1.20	11.80	1.81×10^{-5}	1.51×10^{-5}
30°C	1.17	11.40	1.86×10^{-5}	1.60×10^{-5}
40°C	1.13	11.10	1.91×10^{-5}	1.69×10^{-5}
50°C	1.09	10.70	1.95×10^{-5}	1.79×10^{-5}
60°C	1.06	10.40	2.00×10^{-5}	1.89×10^{-5}
70°C	1.03	10.10	2.04×10^{-5}	1.99×10^{-5}
80°C	1.00	9.81	2.09×10^{-5}	2.09×10^{-5}
90°C	0.97	9.54	2.13×10^{-5}	2.19×10^{-5}
100°C	0.95	9.28	2.17×10^{-5}	2.29×10^{-5}
120°C	0.90	8.82	2.26×10^{-5}	2.51×10^{-5}
140°C	0.85	8.38	2.34×10^{-5}	2.74×10^{-5}
160°C	0.81	7.99	2.42×10^{-5}	2.97×10^{-5}
180°C	0.78	7.65	2.50×10^{-5}	3.20×10^{-5}
200°C	0.75	7.32	2.57×10^{-5}	3.44×10^{-5}
	slugs/ft³	lbf/ft³	lbf-s/ft²	ft²/s
0°F	0.00269	0.0866	3.39×10^{-7}	1.26×10^{-4}
20°F	0.00257	0.0828	3.51×10^{-7}	1.37×10^{-4}
40°F	0.00247	0.0794	3.63×10^{-7}	1.47×10^{-4}
60°F	0.00237	0.0764	3.74×10^{-7}	1.58×10^{-4}
80°F	0.00228	0.0735	3.85×10^{-7}	1.69×10^{-4}
100°F	0.00220	0.0709	3.96×10^{-7}	1.80×10^{-4}
120°F	0.00213	0.0685	4.07×10^{-7}	1.91×10^{-4}
150°F	0.00202	0.0651	4.23×10^{-7}	2.09×10^{-4}
200°F	0.00187	0.0601	4.48×10^{-7}	2.40×10^{-4}
300°F	0.00162	0.0522	4.96×10^{-7}	3.05×10^{-4}
400°F	0.00143	0.0462	5.40×10^{-7}	3.77×10^{-4}

Table A.5 APPROXIMATE PHYSICAL PROPERTIES OF WATER* AT ATMOSPHERIC PRESSURE

Temperature	Density	Specific Weight	Dynamic Viscosity	Kinematic Viscosity	Vapor Pressure
	kg/m³	N/m³	N · s/m²	m²/s	N/m² abs
0°C	1000	9810	1.79×10^{-3}	1.79×10^{-6}	611
5°C	1000	9810	1.51×10^{-3}	1.51×10^{-6}	872
10°C	1000	9810	1.31×10^{-3}	1.31×10^{-6}	1,230
15°C	999	9800	1.14×10^{-3}	1.14×10^{-6}	1,700
20°C	998	9790	1.00×10^{-3}	1.00×10^{-6}	2,340
25°C	997	9781	8.91×10^{-4}	8.94×10^{-7}	3,170
30°C	996	9771	7.97×10^{-4}	8.00×10^{-7}	4,250
35°C	994	9751	7.20×10^{-4}	7.24×10^{-7}	5,630
40°C	992	9732	6.53×10^{-4}	6.58×10^{-7}	7,380
50°C	988	9693	5.47×10^{-4}	5.53×10^{-7}	12,300
60°C	983	9643	4.66×10^{-4}	4.74×10^{-7}	20,000
70°C	978	9594	4.04×10^{-4}	4.13×10^{-7}	31,200
80°C	972	9535	3.54×10^{-4}	3.64×10^{-7}	47,400
90°C	965	9467	3.15×10^{-4}	3.26×10^{-7}	70,100
100°C	958	9398	2.82×10^{-4}	2.94×10^{-7}	101,300
	slugs/ft³	lbf/ft³	lbf-s/ft²	ft²/s	psia
40°F	1.94	62.43	3.23×10^{-5}	1.66×10^{-5}	0.122
50°F	1.94	62.40	2.73×10^{-5}	1.41×10^{-5}	0.178
60°F	1.94	62.37	2.36×10^{-5}	1.22×10^{-5}	0.256
70°F	1.94	62.30	2.05×10^{-5}	1.06×10^{-5}	0.363
80°F	1.93	62.22	1.80×10^{-5}	0.930×10^{-5}	0.506
100°F	1.93	62.00	1.42×10^{-5}	0.739×10^{-5}	0.949
120°F	1.92	61.72	1.17×10^{-5}	0.609×10^{-5}	1.69
140°F	1.91	61.38	0.981×10^{-5}	0.514×10^{-5}	2.89
160°F	1.90	61.00	0.838×10^{-5}	0.442×10^{-5}	4.74
180°F	1.88	60.58	0.726×10^{-5}	0.385×10^{-5}	7.51
200°F	1.87	60.12	0.637×10^{-5}	0.341×10^{-5}	11.53
212°F	1.86	59.83	0.593×10^{-5}	0.319×10^{-5}	14.70

* Notes: (1) Bulk modulus E_v of water is approximately 2.2 GPa (3.2×10^5 psi); (2) water–air surface tension is approximately 7.3×10^{-2} N/m (5×10^{-3} lbf/ft) from 10°C to 50°C.

From Engineering Fluid Mechanics of Crowe et al, 2009

Effect of Re on the pressure distribution on surface of circle

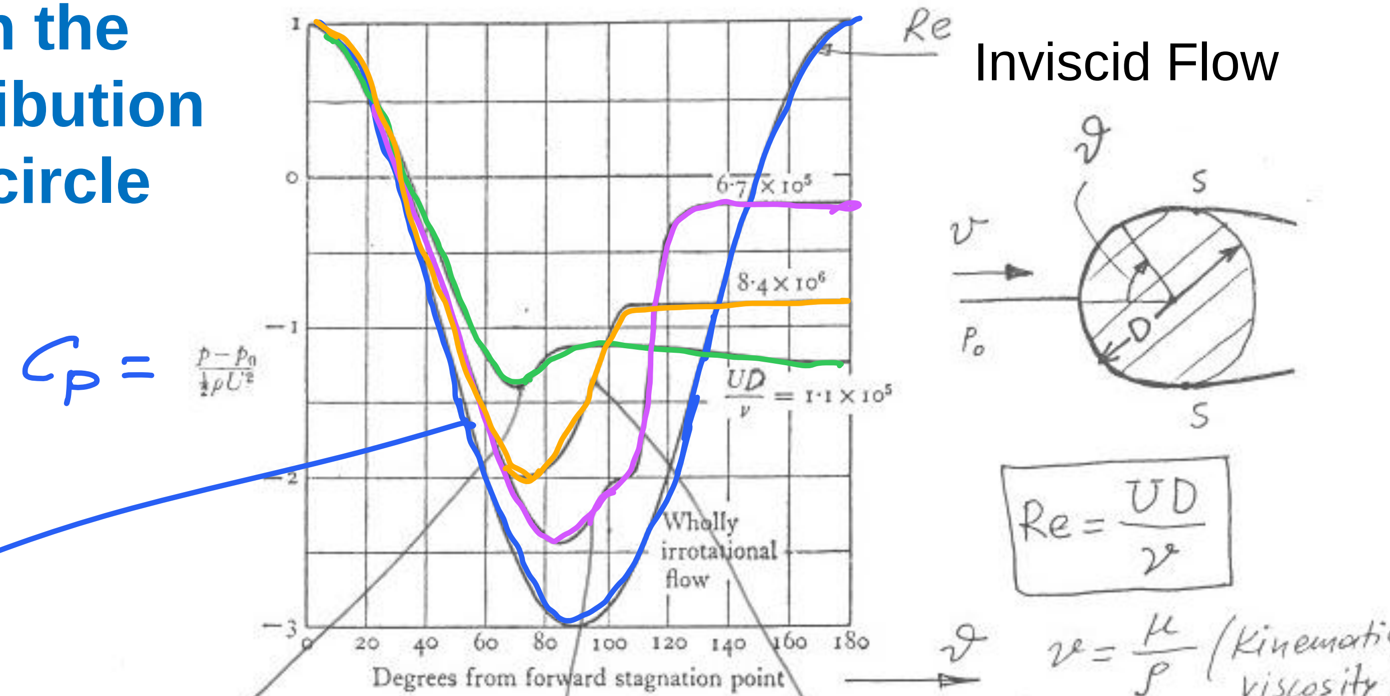
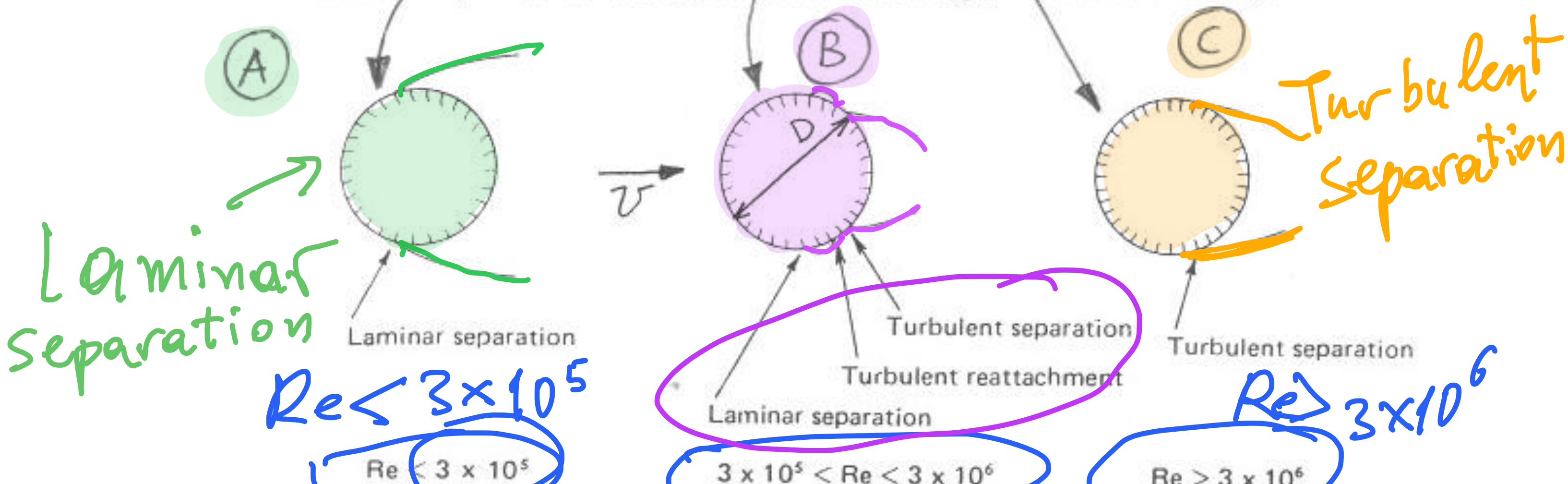
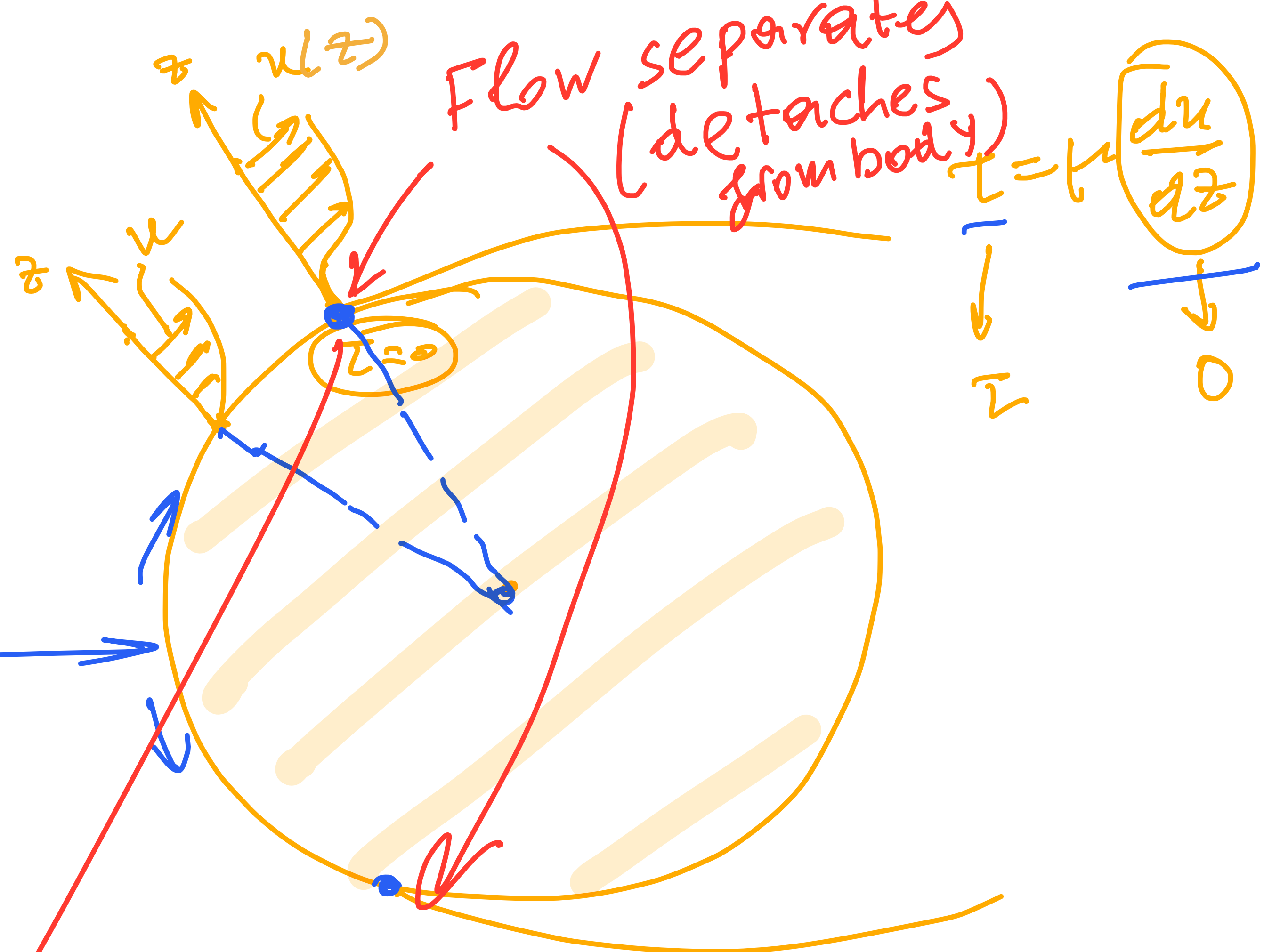


Figure 5.11.5. The measured pressure distribution at the surface of a circular cylinder in a stream of speed U at different Reynolds numbers; p_0 = pressure at infinity.



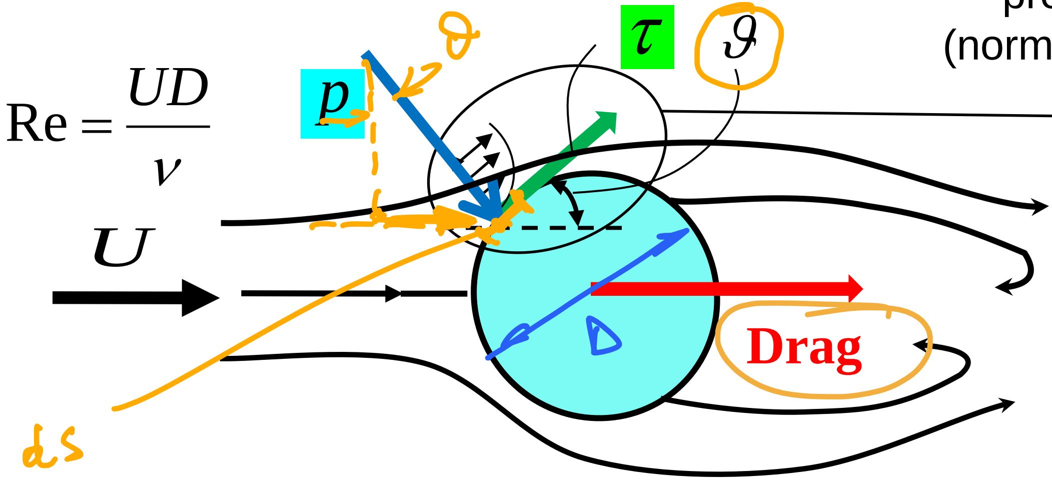


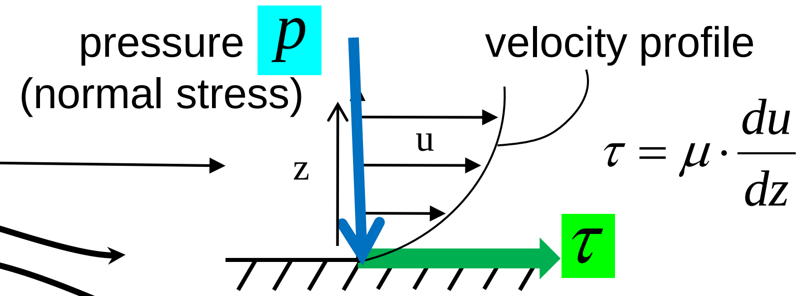
see also "Calculated Flow Around Cylinder in Uniform Flow" in the OG website!

criterion for separation:

$$\tau = \mu \frac{du}{dz} = 0$$

Drag and Drag coefficient





μ=dynamic viscosity

τ= friction (shear stress) acting on the body

Total Drag = Friction Drag + Pressure Drag

$$\text{Friction Drag} = \int_{\text{body}} \tau\, ds\, \cos\vartheta \neq 0$$

$$\text{Pressure (or form) Drag} = \int_{\text{body}} p\, ds\, \sin\vartheta \neq 0$$

Drag Coefficient (in 2-D):

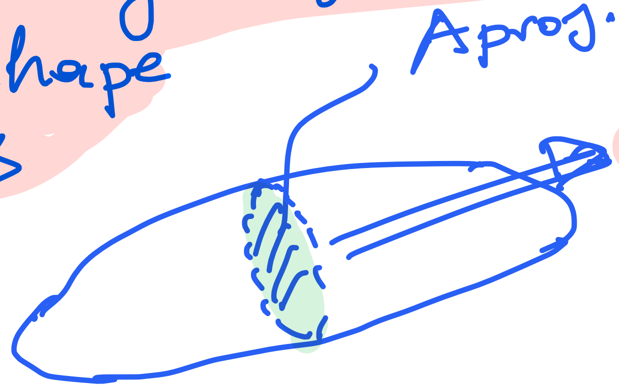
$$C_D = \frac{\text{Drag force per unit width}}{\frac{1}{2}\rho U^2 D}$$

(in 3-D):

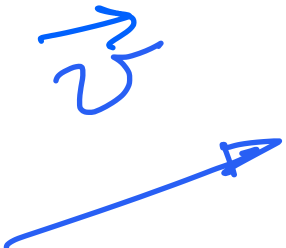
$$C_D = \frac{\text{Drag force}}{\frac{1}{2}\rho U^2 A_{proj}}$$

A_{proj} is the **projected area** of the body on a plane normal to the direction of inflow

Definition of Drag coeff. C_D for general shape bodies



(Force in the same direction as $\vec{v}$)



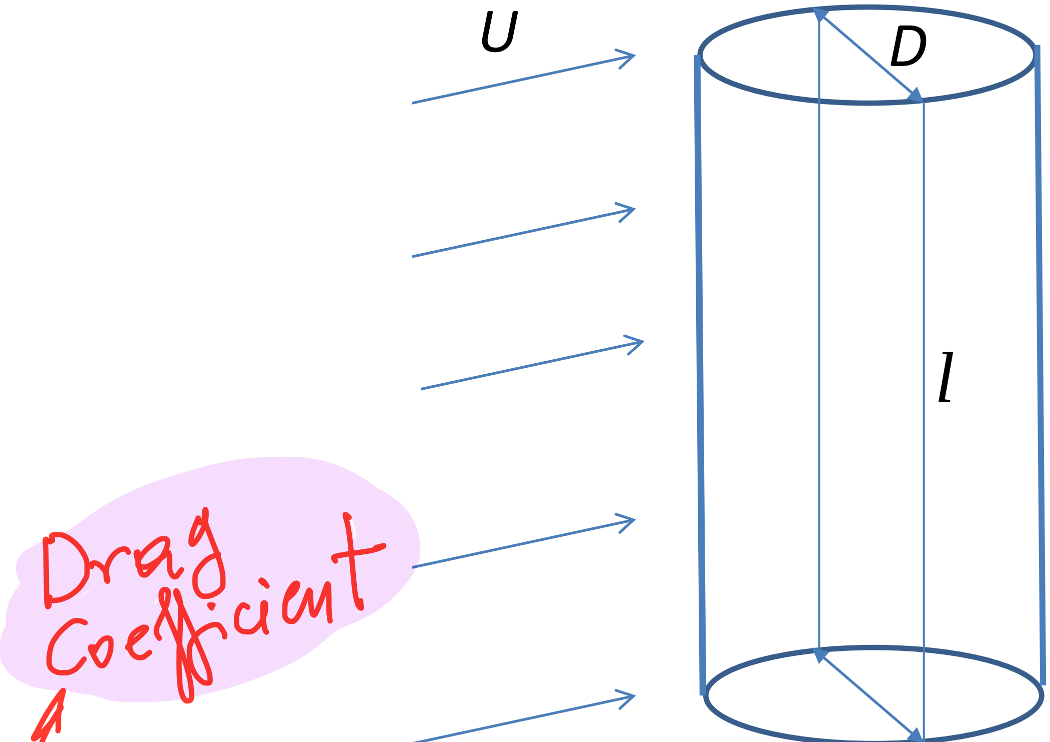
$$C_D = \frac{\text{Drag}}{\frac{1}{2}\rho v^2 A_{proj.}}$$

ρ → density of fluid

$A_{proj.}$ = Projected Area of object on a plane which is perpendicular to inflow direction $\vec{U}$

* C_D is unitless!

Drag force on a cylinder subject to uniform current *U*



$$A_{proj} = Dl$$

Drag force =

$= C_D \frac{1}{2} \rho U^2 \cdot A_{proj.}$

$$C_D = \frac{\text{Drag force}}{\frac{1}{2}\rho U^2 A_{proj}} = \frac{\text{Drag force}}{\frac{1}{2}\rho U^2 Dl} = \frac{\text{Drag force per unit width}}{\frac{1}{2}\rho U^2 D}$$

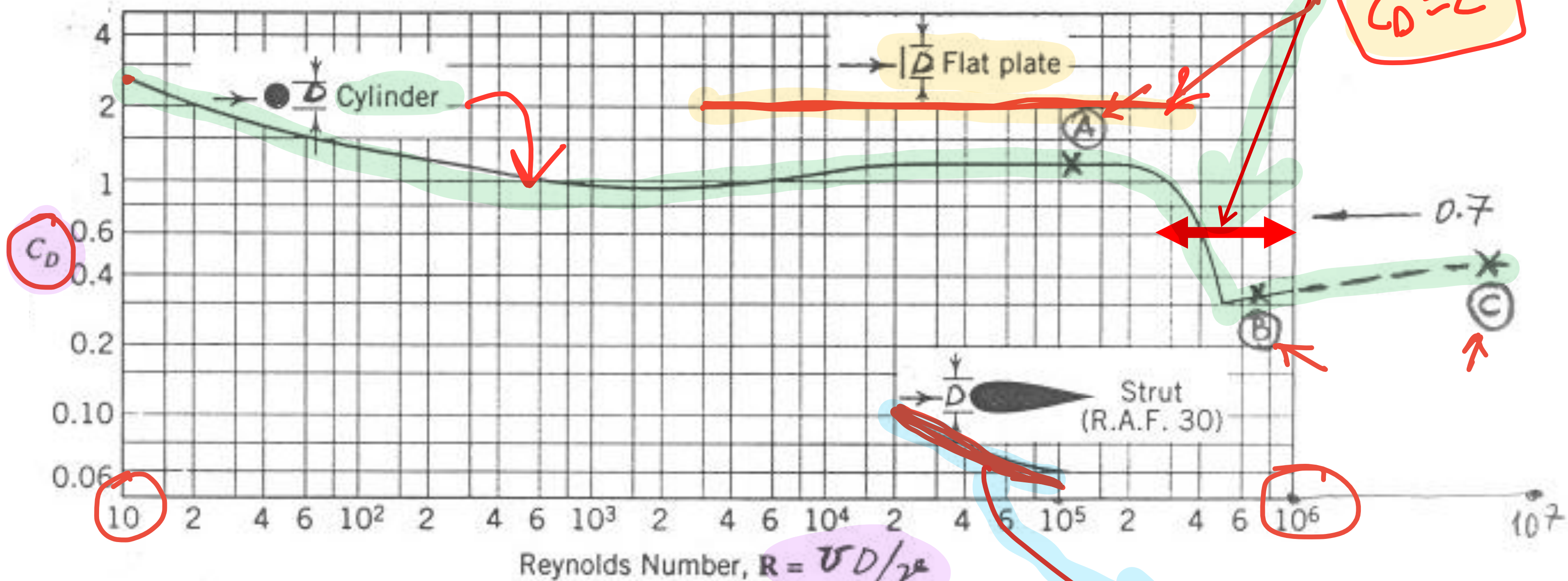
Effect of Re on Drag coefficient on Cylinder

Drag↓

Drag "crisis"

For flat plate

$C_D = 2$



significantly smaller C_D for "streamlined" body

$$C_D = \frac{\text{Drag force per unit width}}{\frac{1}{2}\rho U^2 D}$$

Note that in the C_D vs Re graph above:

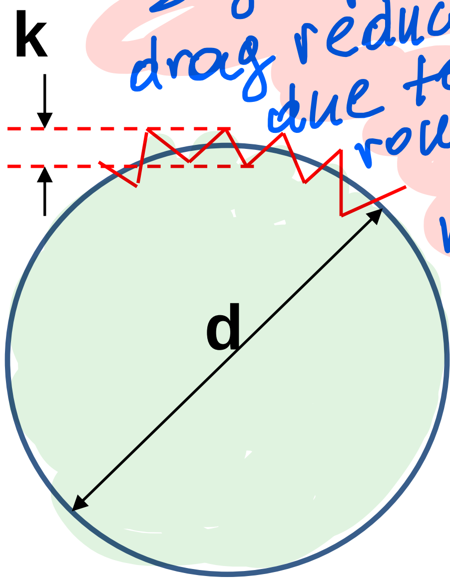
→ A has the largest C_D due to larger extent of separation zone as shown in the flow pattern a few slides above

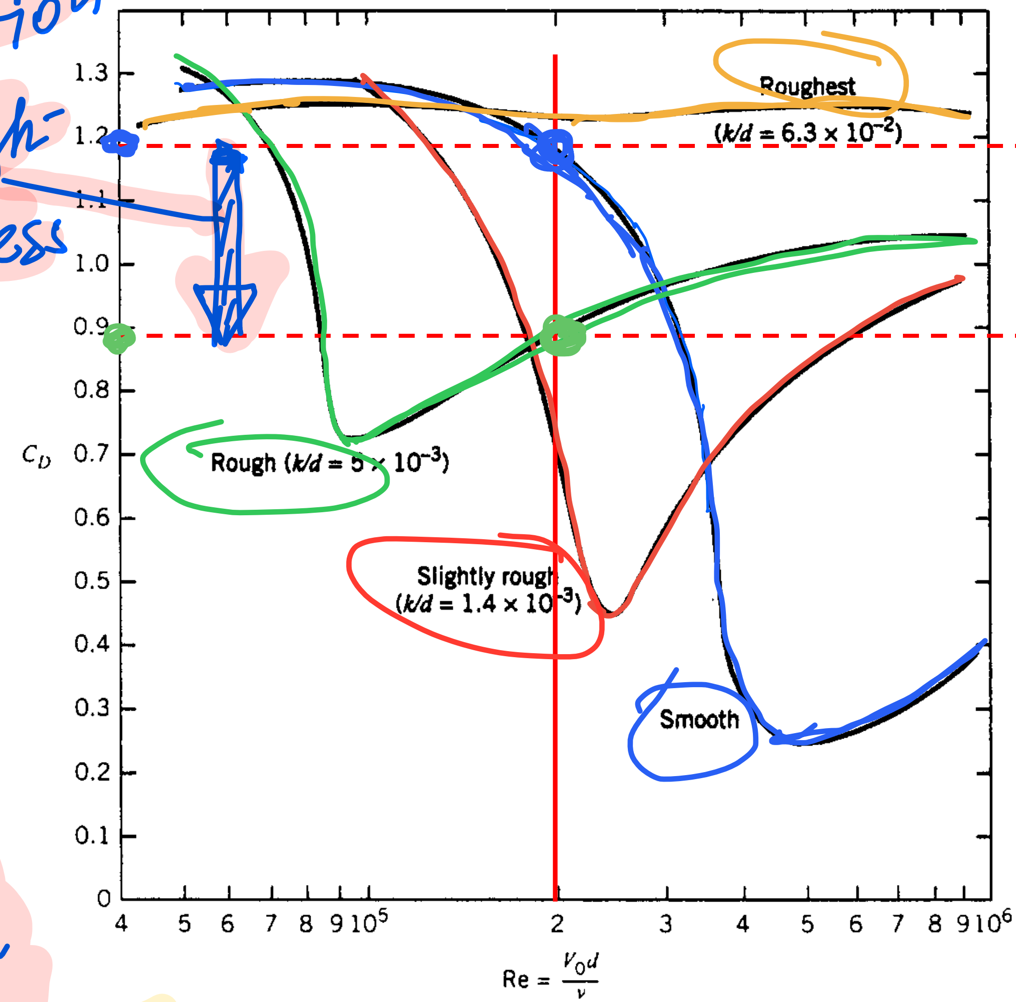
→ B has the smallest C_D due to smaller extent of separation. However this condition is not recommended due to sharp change of C_D as speed/inflow changes

→ C is the best choice since turbulent flow reduces separation zone ⇒ $C_D \downarrow$

Effect of roughness on Drag coefficient on Cylinder

significant drag reduction due to roughness





$C_{D,smooth}$

$C_{D,rough}$

k/d= relative roughness

Roughness turns flow turbulent and REDUCES Drag

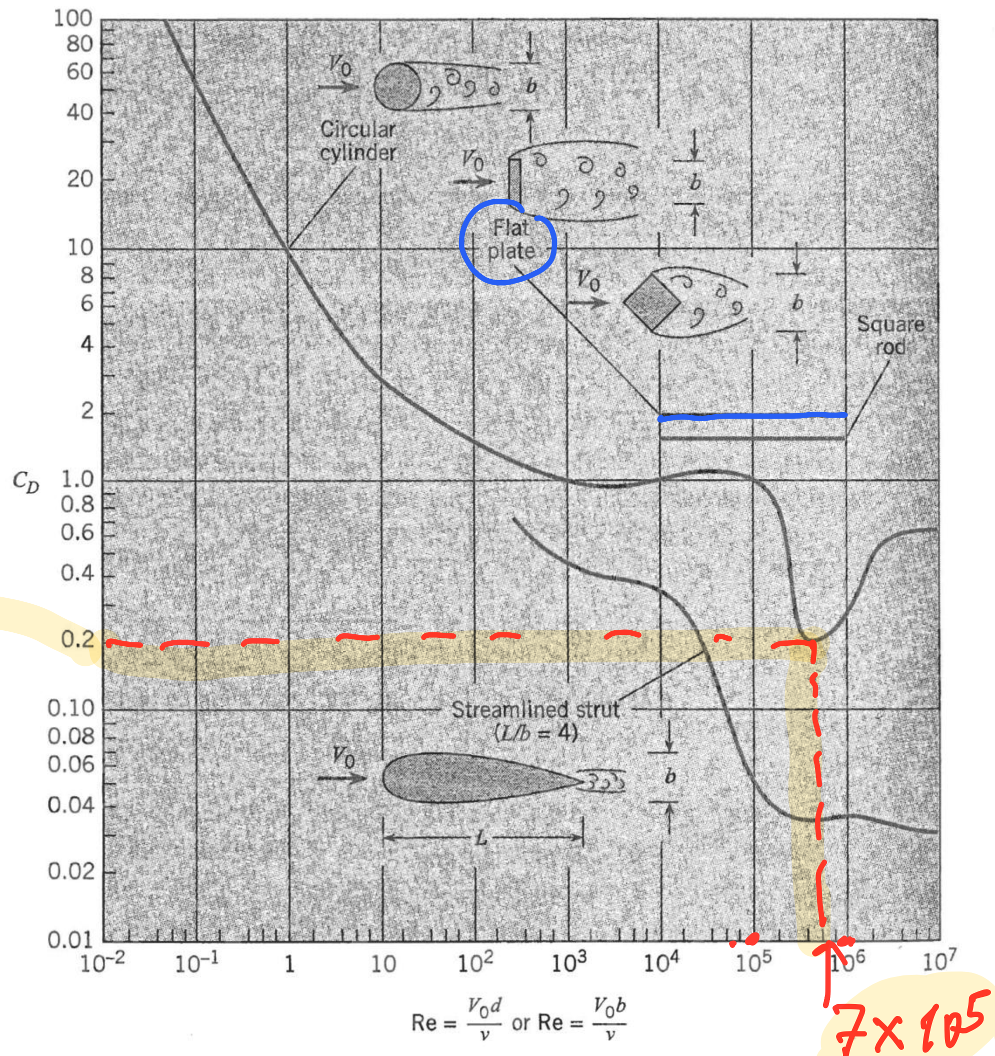
From Engineering Fluid Mechanics of Crowe et al, 2009

pits in golfball trigger turbulence

Drag coefficients for some other 2-D shapes

FIGURE 11.5

Coefficient of drag versus Reynolds number for two-dimensional bodies. [Data sources: Bullivant (5), Defoe (9), Goett and Bullivant (12), Jacobs (15), Jones (17), and Lindsey (21)]

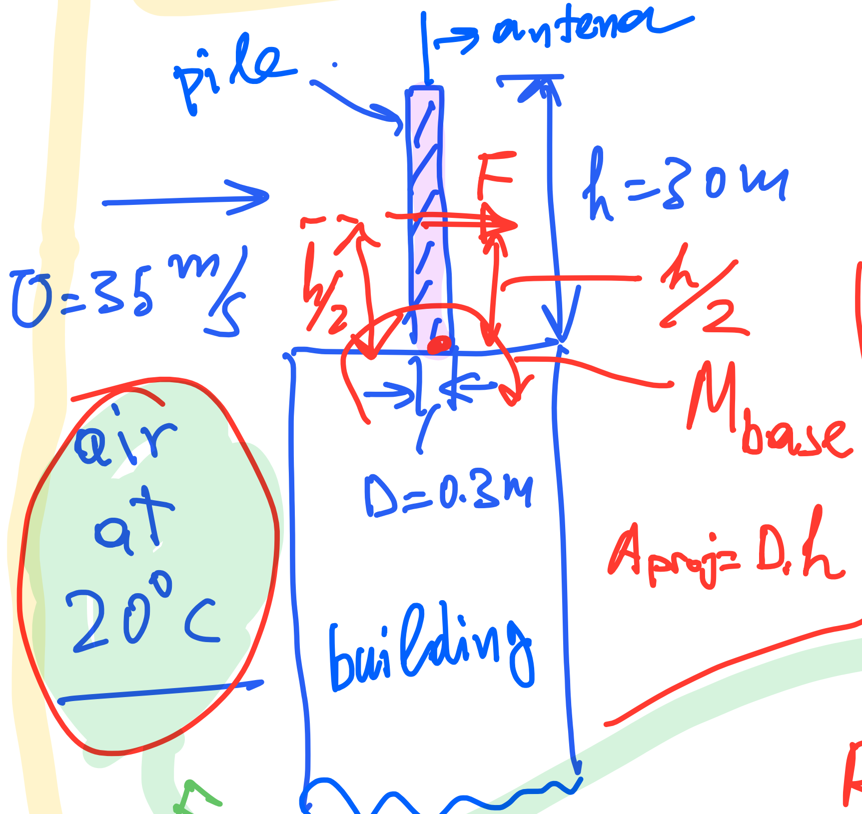


Graph used in example problem in next slide

From Engineering Fluid Mechanics of Crowe et al, 2009

Ex. 11.1 (p. 486) from Crowe's book (exerpts are provided on class website)

Find Force acting on pile and moment exerted at its base.



$$F = \frac{1}{2}\rho V^2 A_{proj.} C_D$$

ρ: $1.2\ \frac{kg}{m^3}$

$$Re = \frac{V.D}{\nu} = \frac{35\frac{m}{s} \times (0.3m)}{1.51\times10^{-5}\frac{m^2}{s}} = 7\times10^5$$

Kinematic viscosity

From Tables for air (see previous slides)

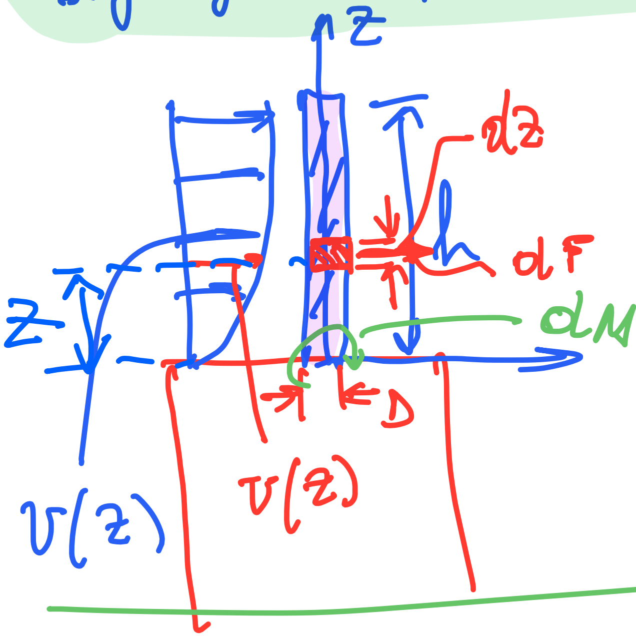
$\Rightarrow C_D = 0.2$

$\Rightarrow \boxed{F = 1{,}323\ N}$

Force applies at the middle of the pile since uniform inflow is assumed.

$$M_{base} = F \times \frac{h}{2} = 19,845 \text{ N-m}$$

If flow not uniform:



$$dF = C_D \frac{1}{2}\rho \left[U(z)\right]^2 . D . dz$$

we assume to be constant

$$F = \int_0^h dF = \int_0^h C_D \frac{1}{2}\rho\, U(z)^2 D . dz = C_D \frac{\rho}{2} D \int_0^h \left[U(z)\right]^2 dz$$

$$dM = z . dF$$

$$M_{base} = \int_0^h dM = \int_0^h z\, dF = C_D \frac{\rho}{2} D \int_0^h z \left[U(z)\right]^2 dz$$