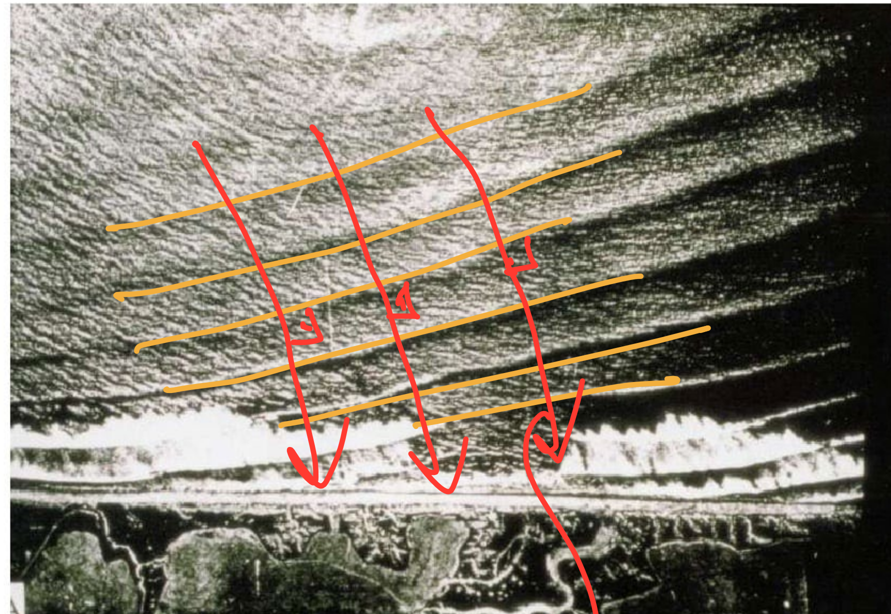
WAVE REFRACTION

Wave Refraction

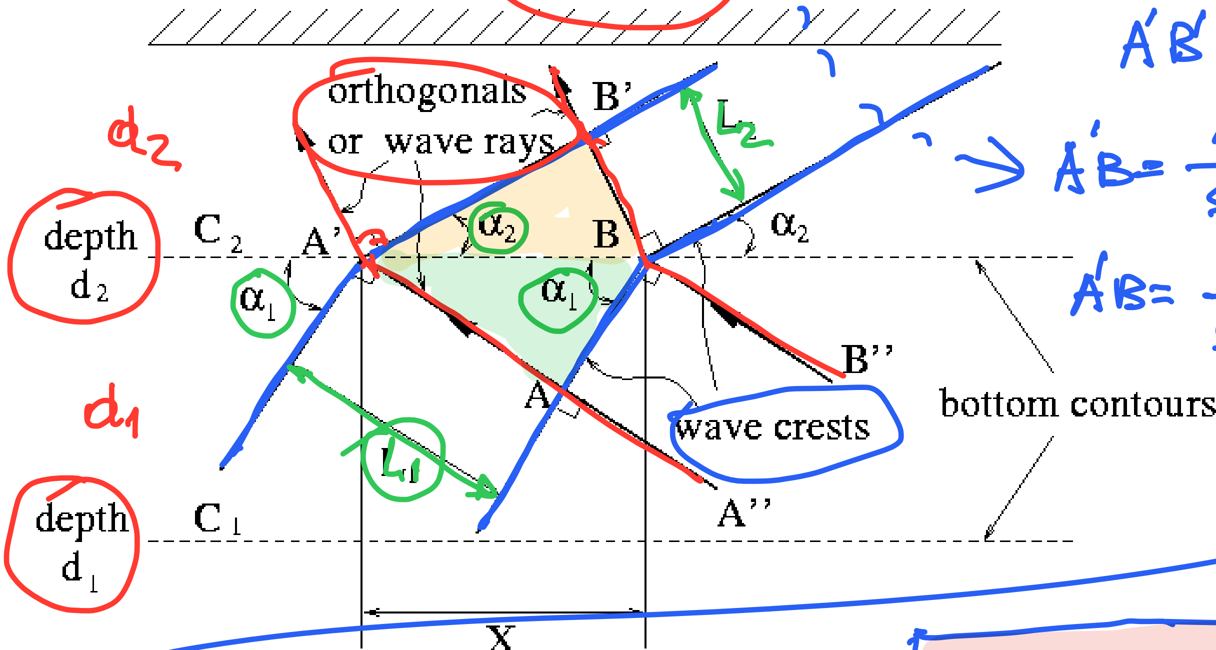


wave crests

wave rays or orthogonal

WAVE REFRACTION

$\sin\alpha_1 = \dfrac{AA'}{A'B}$



From orthogonal triangles $A'B'B$ & $A'AB$

$$\Rightarrow A'B = \frac{AA'}{\sin\alpha_1} = \frac{L_1}{\sin\alpha_1}$$

$$A'B = \frac{BB'}{\sin\alpha_2} = \frac{L_2}{\sin\alpha_2}$$

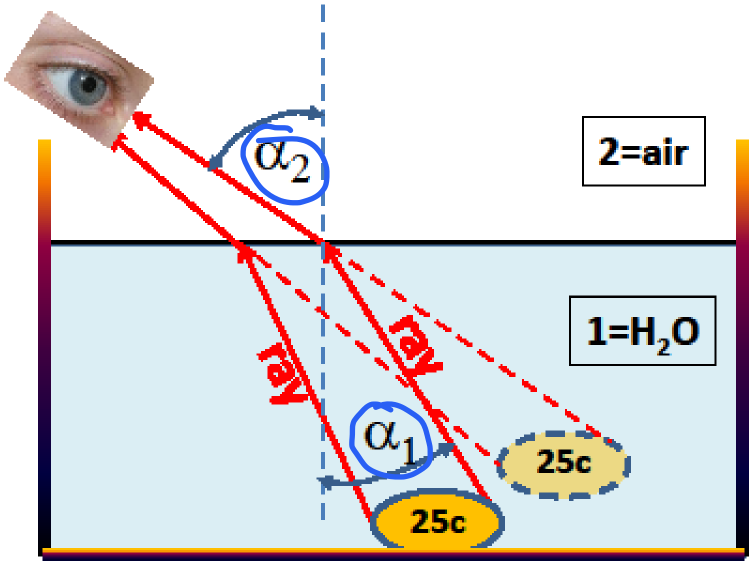
$$\frac{L_1}{\sin\alpha_1} = \frac{L_2}{\sin\alpha_2} \Longrightarrow \boxed{\frac{\sin\alpha_1}{\sin\alpha_2} = \frac{L_1}{L_2} = \frac{C_1}{C_2}}$$

Snell's Law

WAVE REFRACTION

Refraction

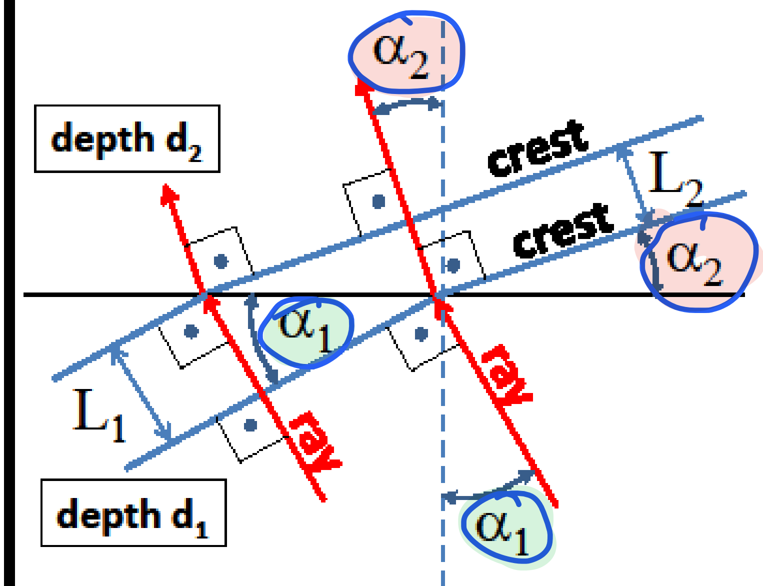
Optics



$C_{H_2O} \approx 0.75 C_{air} \Rightarrow C_2 > C_1 \Rightarrow \alpha_2 > \alpha_1$

Refraction of optical rays due to different speed of light from water to air!

Wave Mechanics

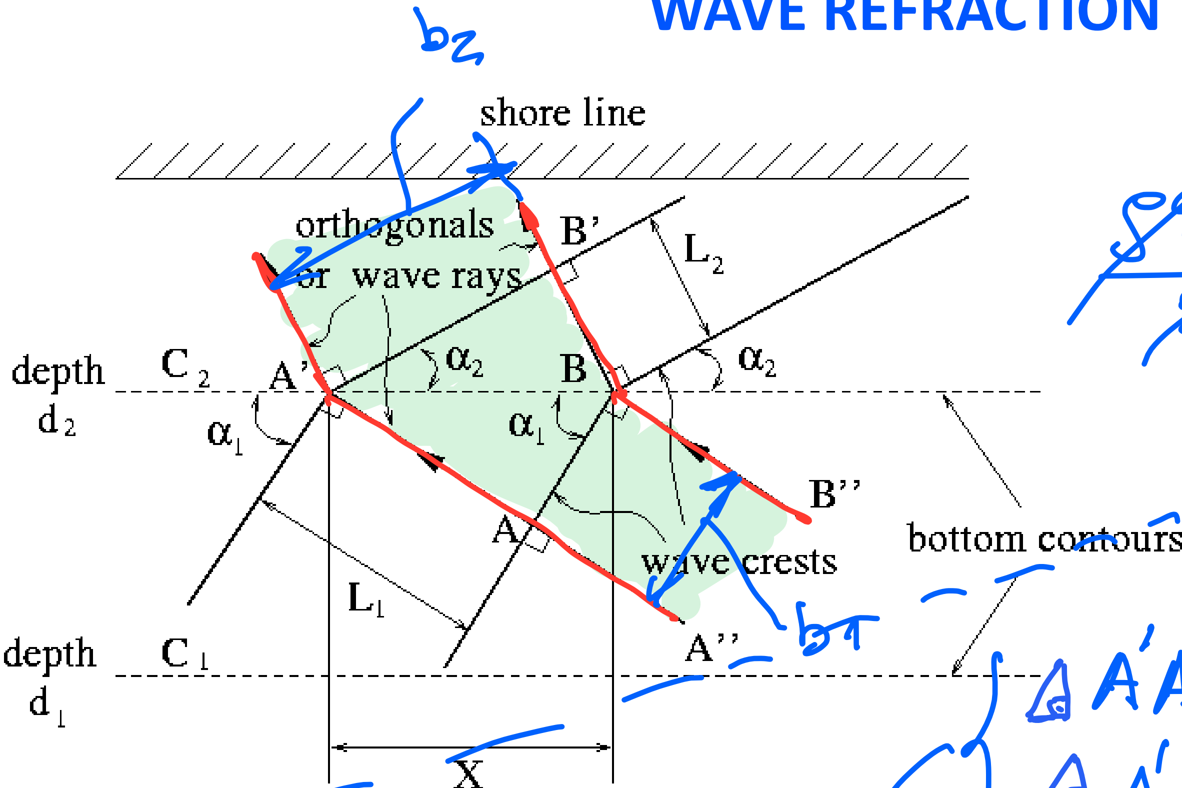


$d_2 < d_1 \Rightarrow C_2 < C_1 \Rightarrow \alpha_2 < \alpha_1$

$$\text{Snell's Law}: \frac{\sin(\alpha_1)}{\sin(\alpha_2)} = \frac{C_1}{C_2} = \frac{L_1}{L_2}$$

Refraction of ocean wave rays due to different wave speeds at different depths.

WAVE REFRACTION



$$P_1 = P_2$$

$$\frac{\cancel{\rho g} H_1^2}{\cancel{8}} C_{g_1} b_1 = \frac{\cancel{\rho g} H_2^2}{\cancel{8}} C_{g_2} b_2$$

$$\boxed{\frac{H_2}{H_1} = \sqrt{\frac{C_{g_1}}{C_{g_2}}} \sqrt{\frac{b_1}{b_2}}}$$

$\triangle A'AB \Rightarrow b_1 = AB = A'B \cos\alpha_1$

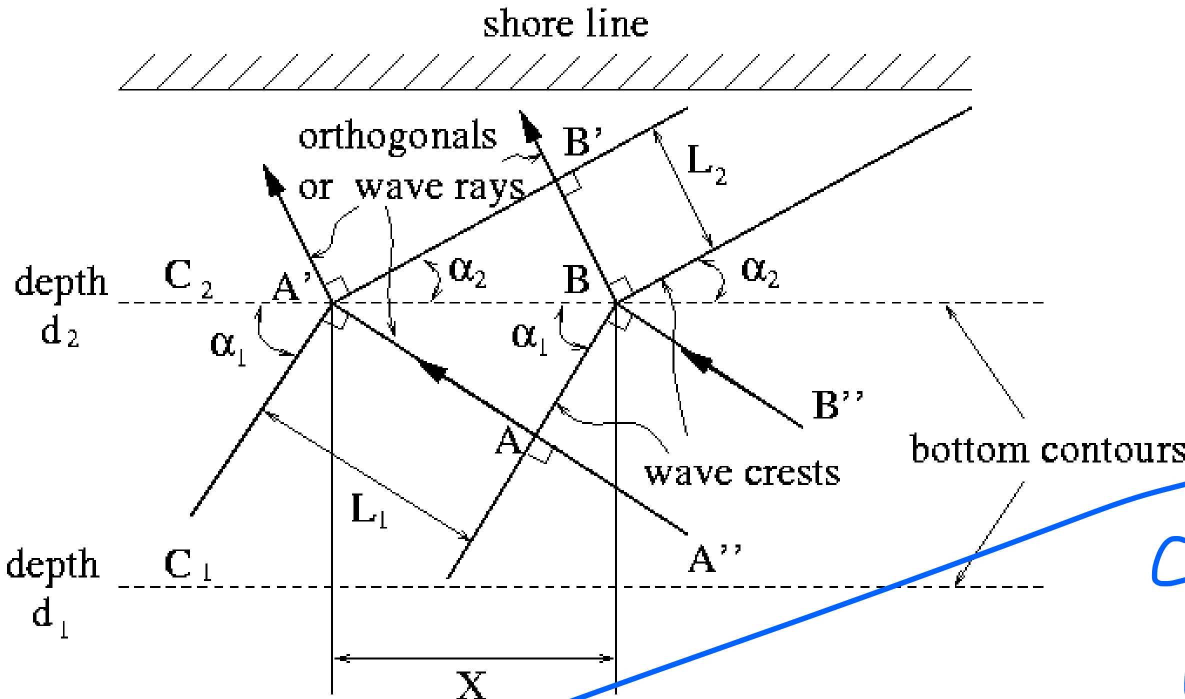
$\triangle A'B'B \Rightarrow b_2 = A'B' = A'B \cos\alpha_2$

$$\boxed{\frac{b_1}{b_2} = \frac{\cos\alpha_1}{\cos\alpha_2}}$$

$$\boxed{\frac{H_2}{H_1} = \sqrt{\frac{C_{g_1}}{C_{g_2}}} \cdot \sqrt{\frac{\cos\alpha_1}{\cos\alpha_2}}}$$

General formu

WAVE REFRACTION



$H_1 \to H_0$ (deep water)

$H_2 \to H$ (at depth d)

$$\boxed{\frac{H}{H_0} = \sqrt{\frac{C_{g0}}{C_g}}\sqrt{\frac{b_0}{b}}}$$

C_{g0}: C_g at deep H_2O

b_0: width at deep H_2O

$$\frac{H}{H_0} = K_s K_R$$

K_R → refraction coeff.

K_s: shealing coeff.

$$= \sqrt{\frac{b_0}{b}} = \sqrt{\frac{\cos\alpha_0}{\cos\alpha}}$$

$$\frac{H}{H_0} = K_s K_R = \frac{H}{H_0'} \cdot \frac{H_0'}{H_0}$$

$\frac{H}{H_0'} = K_s$, $\frac{H_0'}{H_0} = K_R$

H_0' = unrefracted deep H_2O height

WAVE REFRACTION - EXAMPLES

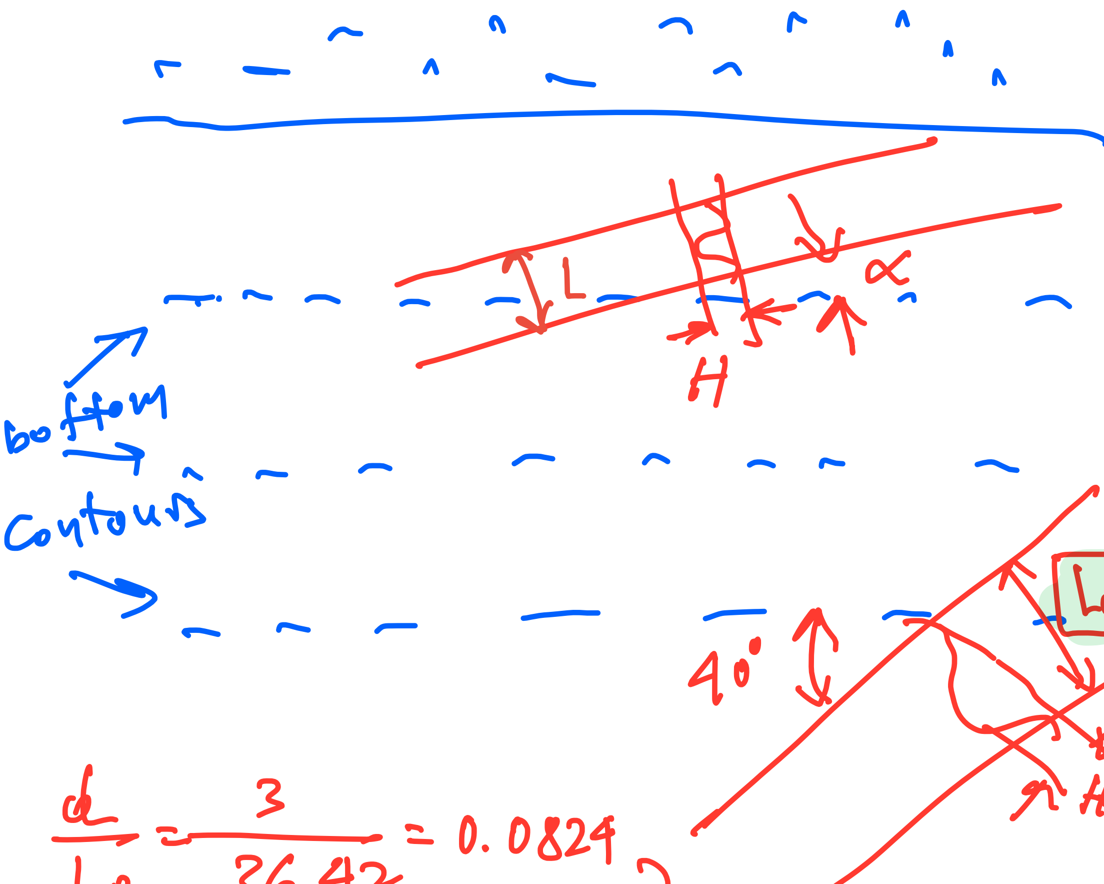
$T = ?$

1. A 4.83 sec plane mono-chromatic wave approaches the beach with its crests in deep water at an angle of 40^o with respect to the straight shoreline, and a wave height of 30 cm. Determine the angle of the crests and the wave height at a depth of $3m$. Consider that the bottom contours are parallel to the shoreline and that the effects of reflection are negligible.

H_o

$\alpha = ?$

$H = ?$



$d = 3m$

Snell's Law

$$\frac{\sin\alpha}{\sin\alpha_o} = \frac{L}{L_o}$$

$$\boxed{L_o} = \frac{gT^2}{2\pi} = \frac{g(4.83)^2}{2\pi} = \boxed{36.42m}$$

$$\frac{d}{L_o} = \frac{3}{36.42} = 0.0824 \xrightarrow{C-1} \frac{d}{L} = 0.1255 \Rightarrow \boxed{L = 23.9m}$$

Table C-1

$$\sin\alpha = \frac{L}{L_0}\sin(40^\circ) \rightsquigarrow \sin\alpha = 0.422 \rightarrow \boxed{\alpha = 25^\circ}$$

$$\frac{H}{H_0} = \underline{K_S}\ \underline{K_R}$$

$$\boxed{K_R} = \sqrt{\frac{b_0}{b}} = \sqrt{\frac{\cos\alpha_0}{\cos\alpha}} =$$

$$= \sqrt{\frac{\cos(40^\circ)}{\cos(25^\circ)}} = \boxed{0.92}$$

$$\frac{H}{H_0'} = \boxed{K_s = 0.951}$$

$$H = H_0 K_S K_R = (0.3m) \times 0.951 \times 0.92 = \boxed{0.2625m}$$

WAVE REFRACTION - EXAMPLES

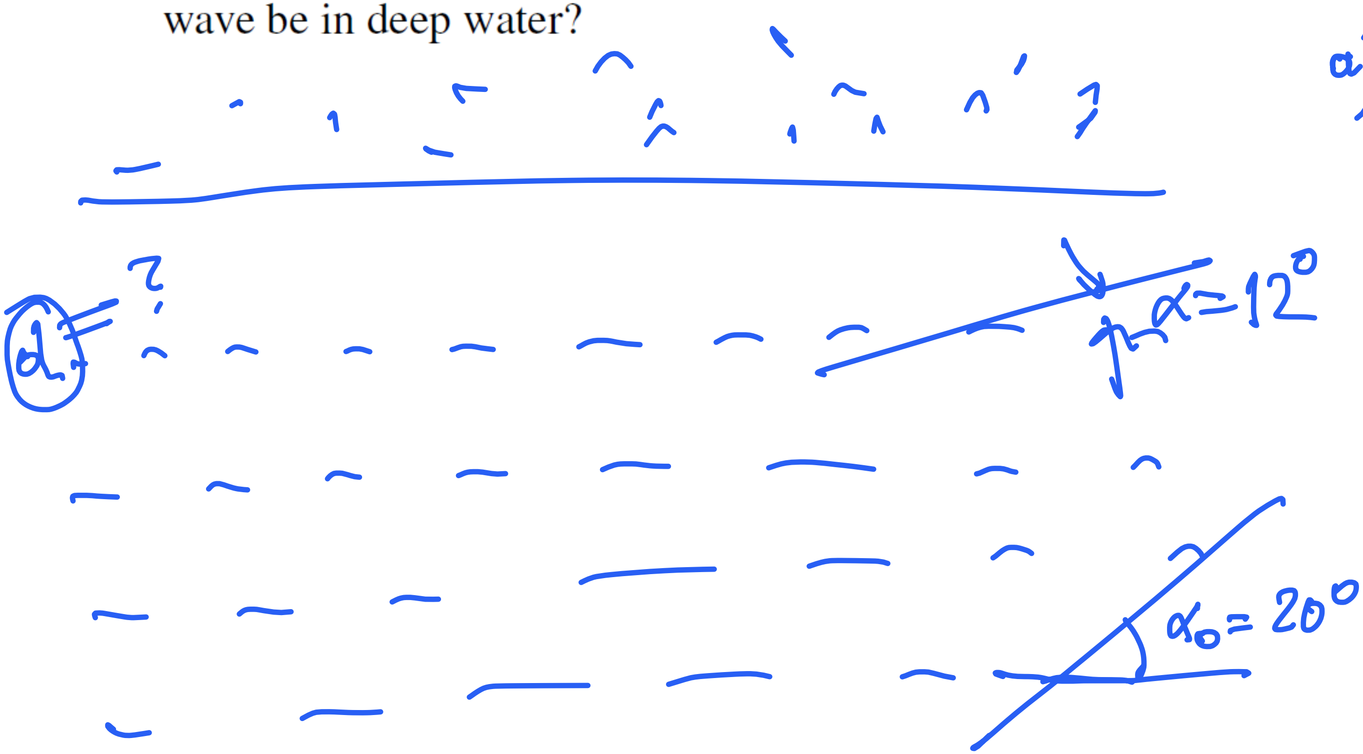
$= T$ $\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad$ $\alpha_0 =$

2. A 4.5 sec sinusoidal wave is approaching the beach with its crests in deep water at an angle of 20^o with respect to the straight shoreline. At a certain distance from the beach the wave crests have been ~~refected~~ frac and form an angle of $\alpha = 12^o$ with respect to the shoreline. Assuming that the bottom contours are parallel to the shoreline and that the effects of reflection are negligible, find the following:

   (a) The wave length at the location where $\alpha = 12^o$

   (b) The depth of the water at the location where $\alpha = 12^o$

   (c) If the wave height at the same depth as that in (b) is 40 cm, what would the height of the wave be in deep water?



a) Snell's Law

$$\frac{\sin\alpha}{\sin\alpha_0} = \frac{L}{L_0}$$

$$L_0 = \frac{gT^2}{2\pi} = 31.6m$$

$$\boxed{L = 19.2m}$$

b) $\boxed{L = L_0 \tanh\left(\frac{2\pi d}{L}\right)}$ ← we got this formula from dispersion relationship in finite depth H_2O

$$\Rightarrow \tanh\left(\frac{2\pi d}{L}\right) = \frac{L}{L_0} = \frac{19.2}{31.6} = 0.608$$

$$\Rightarrow \frac{2\pi d}{L} = \tanh^{-1}(0.608) = 0.7057 \Rightarrow \boxed{d = 2.16m}$$

$$\rightarrow \frac{d}{L} = \frac{2.16}{19.2} = 0.113 > 0.04 \quad \text{Transitional}$$

c) $H = 0.4m \qquad H_0 = ?$

$$\frac{H}{H_0} = K_S K_R$$

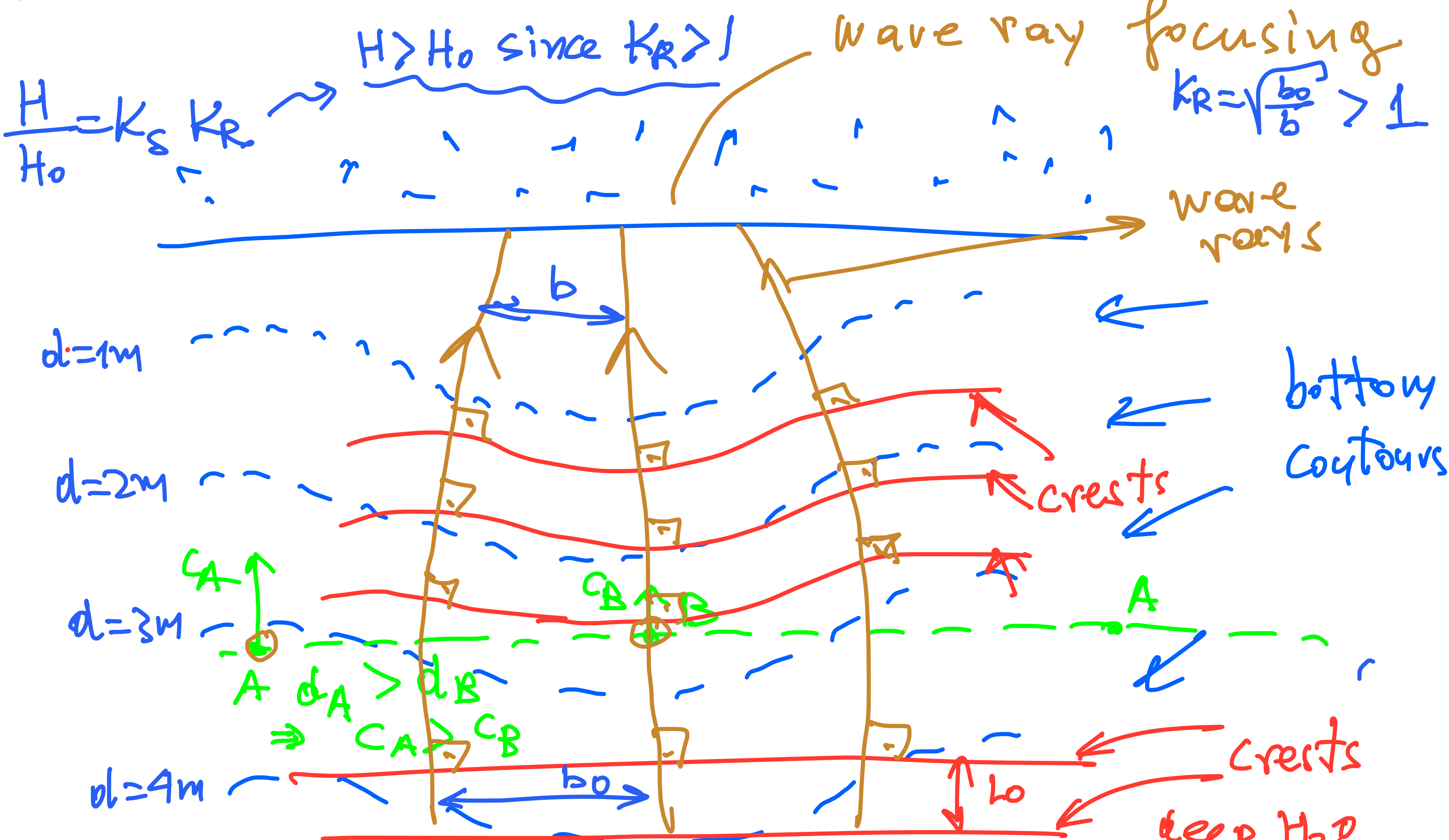
K_S from C-1 and $\frac{d}{L} = \frac{2.16}{19.2} = 0.113 \xRightarrow{C-1} K_S = 0.9752$
$\left(C\left(=\frac{H}{H_0'}\right)\right)$

$$K_R = \sqrt{\frac{\cos\alpha_0}{\cos\alpha}} = \sqrt{\frac{\cos(20°)}{\cos(12°)}} = 0.98 \; ; \; \frac{H}{H_0} = 0.9732 \times 0.98 = 0.956 \Rightarrow$$

$$\Rightarrow \boxed{H_0 = 0.418m}$$

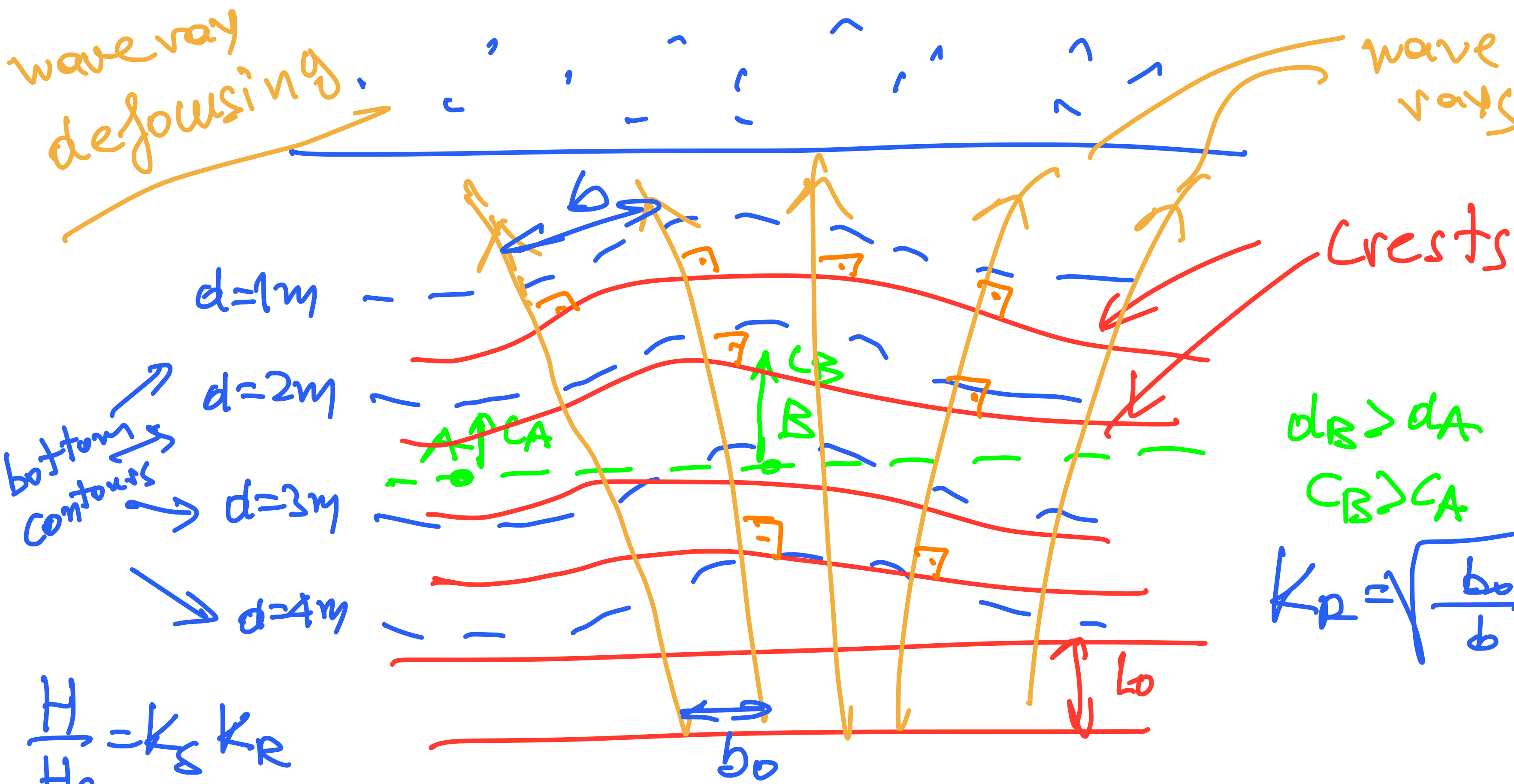
WAVE REFRACTION - EXAMPLES

Submarine ridge



WAVE REFRACTION - EXAMPLES

Submarine canyon



$d_B > d_A$

$C_B > C_A$

$$K_R = \sqrt{\frac{b_o}{b}} < 1$$

$$\frac{H}{H_0} = K_S K_R$$

$H < H_0$ since $K_R < 1$

WAVE REFRACTION - EXAMPLES

INTRODUCTION 15

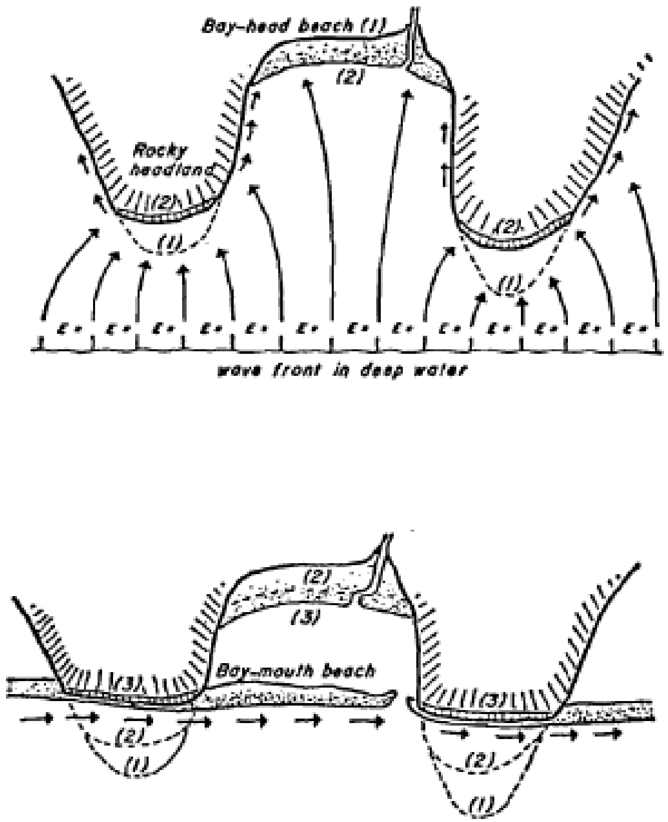


FIG. 6. Waves straighten a rocky coast. *Top:* Zones of equal wave energy in deep water are concentrated by wave refraction so that headlands are attacked. *Bottom:* Eventually headlands are cut back and furnish enough sand to build a straight continuous beach.

5. For the image below of a headland with wave refraction around it qualititatively explain what the underlying depth contours must be and why the wave crests are bending as they do. Draw the ray lines. Where is wave energy more and less concentrated.

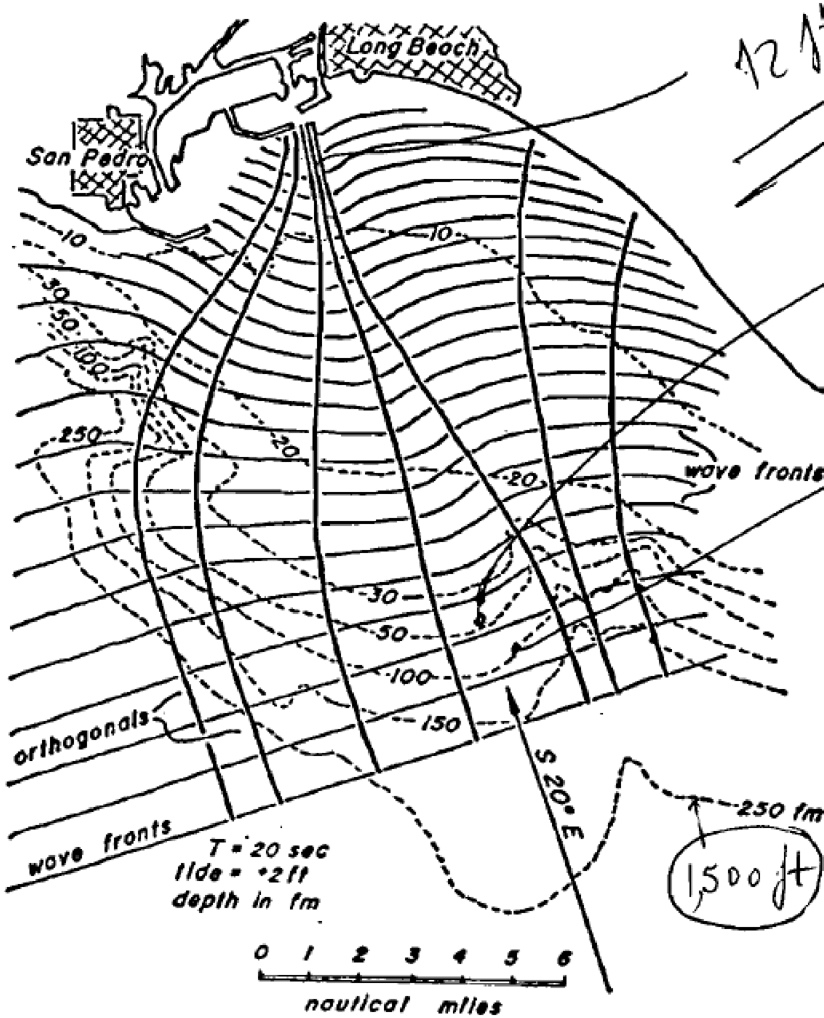
WAVE REFRACTION - EXAMPLES

84 WAVES AND BEACHES

12 ft high

$50 \times 6 = 300$ ft

600 ft



$\boxed{1 \text{ fm} = 6 \text{ ft}}$

fathom

1,500 ft

$L_0 = 2{,}000$ ft

FIG. 30. Refraction diagram for destructive waves at Long Beach, California, showing how underwater topography several hundred feet deep and a dozen miles offshore focused wave energy on the breakwater.