From 10th Edition of Textbook *

Water (10°C, 1 atm, Table A.5) $\gamma_{water} = 9810 \text{ N/m}^3$.
Oil. $\gamma_{oil} = S\gamma_{water, 4°C} = 0.8(9810 \text{ N/m}^3) = 7850 \text{ N/m}^3$.

State the Goal

p_3 (kPa gage) ⬅ pressure at bottom of the tank

Generate Ideas and Make a Plan

Because the goal is p_3, apply the hydrostatic equation to the water. Then, analyze the oil. The plan steps are

1. Find p_2 by applying the hydrostatic equation (3.10a).
2. Equate pressures across the oil–water interface.
3. Find p_3 by applying the hydrostatic equation given in Eq. (3.10a).

Solution

1. Hydrostatic equation (oil)

$$\frac{p_1}{\gamma_{oil}} + z_1 = \frac{p_2}{\gamma_{oil}} + z_2$$

$$\frac{0 \text{ Pa}}{\gamma_{oil}} + 3 \text{ m} = \frac{p_2}{0.8 \times 9810 \text{ N/m}^3} + 2.1 \text{ m}$$

$$p_2 = 7.063 \text{ kPa}$$

2. Oil–water interface

$$p_2|_{oil} = p_2|_{water} = 7.063 \text{ kPa}$$

3. Hydrostatic equation (water)

$$\frac{p_2}{\gamma_{water}} + z_2 = \frac{p_3}{\gamma_{water}} + z_3$$

$$\frac{7.063 \times 10^3 \text{ Pa}}{9810 \text{ N/m}^3} + 2.1 \text{ m} = \frac{p_3}{9810 \text{ N/m}^3} + 0 \text{ m}$$

$$\boxed{p_3 = 27.7 \text{ kPa gage}}$$

Review

Validation: Because oil is less dense than water, the answer should be slightly smaller than the pressure corresponding to a water column of 3 m. From Table F.1, a water column of 10 m ≈ 1 atm. Thus, a 3 m water column should produce a pressure of about 0.3 atm = 30 kPa. The calculated value appears correct.

Pressure Variation in the Atmosphere

This subsection describes how to calculate pressure, density and temperature in the atmosphere for applications such as modeling of atmospheric dynamics and the design of gliders, airplanes, balloons, and rockets.

Equations for pressure variation in the earth's atmosphere are derived by integrating the hydrostatic differential equation (3.7). To begin the derivation, write the ideal gas law (2.5):

$$\rho = \frac{p}{RT} \qquad \text{I.G.L.} \tag{3.14}$$

Multiply by g:

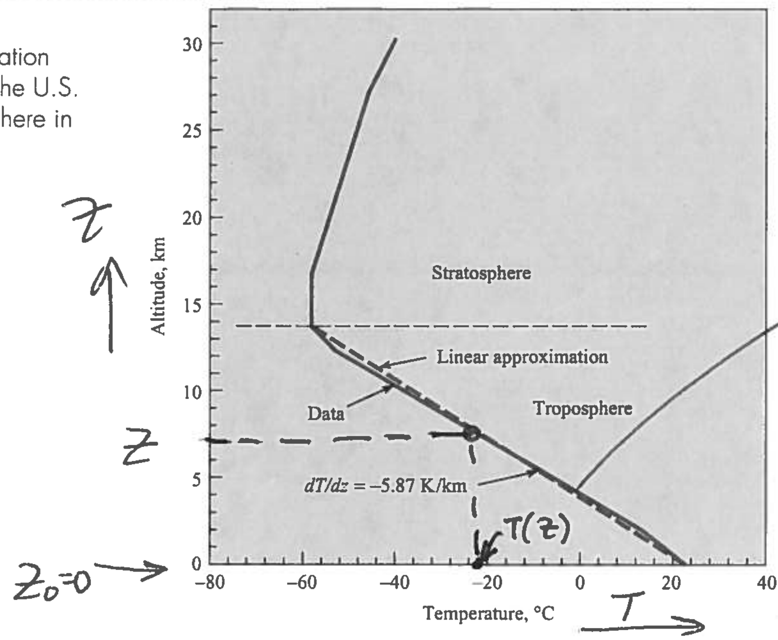
$$\gamma = \frac{pg}{RT} \tag{3.15}$$

Equation (3.15) requires temperature-versus-elevation data for the atmosphere. It is common practice to use the U.S. Standard Atmosphere (1). The **U.S. Standard Atmosphere** defines values for atmospheric temperature, density, and pressure over a wide range of altitudes. The first model was published in 1958; this was updated in 1962, 1966, and 1976. The U.S. Standard Atmosphere gives average conditions over the United States at 45° N latitude in July.

The U.S. Standard Atmosphere also gives average conditions at sea level. The sea level temperature is 15°C (59°F), the pressure is 101.33 kPa abs (14.696 psia), and the density is 1.225 kg/m³ (0.002377 slugs/ft³).

Temperature data for the U.S. Standard Atmosphere are given in Fig. 3.9 for the lower 30 km of the atmosphere. The atmosphere is about 1000 km thick and is divided into five layers, so Fig. 3.9 only gives data near the earth's surface. In the **troposphere**, defined as the

FIGURE 3.9
Temperature variation with altitude for the U.S. standard atmosphere in July (1).



$T(z)$

$$\alpha = 5.87 \frac{°K}{km} = 10.57 \frac{°F}{km}$$

layer between sea level and 13.7 km (45,000 ft), the temperature decreases nearly linearly with increasing elevation at a lapse rate of 5.87 K/km. The **stratosphere** is the layer that begins at the top of the troposphere and extends up to about 50 km. In the lower regions of the stratosphere, the temperature is constant at −57.5°C, to an altitude of 16.8 km (55,000 ft), and then the temperature increases monotonically to −38.5°C at 30.5 km (100,000 ft).

Pressure Variation in the Troposphere

$$\frac{dT}{dz} = -\alpha \Rightarrow dT = -\alpha\, dz \Rightarrow T\Big|_{z_0}^{z} = -\alpha\, z\Big|_{z_0}^{z} \Rightarrow$$

Let the temperature T be given by

$$T = T_0 - \alpha(z - z_0) \tag{3.16}$$

In this equation T_0 is the temperature at a reference level where the pressure is known, and α is the lapse rate. Combine Eq. (3.15) with the hydrostatic differential equation (3.7) to give

$$\frac{dp}{dz} = -\gamma \Rightarrow \frac{dp}{dz} = -\frac{pg}{RT} \tag{3.17}$$

Math reminders

- $\int \frac{dx}{x} = \ln|x|$
- $\ln A - \ln B = \ln\left(\frac{A}{B}\right)$
- $n \ln A = \ln A^n$

Substituting Eq. (3.16) into Eq. (3.17) gives

$$\frac{dp}{dz} = -\frac{pg}{R[T_0 - \alpha(z - z_0)]} \Rightarrow \frac{dp}{p} = -\frac{g\,dz}{R[T_0 - \alpha(z - z_0)]}$$

$$\Rightarrow \frac{dp}{p} = +\frac{g\, \overbrace{d[T_0 - \alpha(z - z_0)]}^{= -dz}}{\alpha R[T_0 - \alpha(z - z_0)]}$$

Separate the variables and integrate to obtain

$$\frac{p}{p_0} = \left[\frac{T_0 - \alpha(z - z_0)}{T_0}\right]^{g/\alpha R}$$

$$\ln p\Big|_{p_0}^{p} = \frac{g}{\alpha R} \ln[T_0 - \alpha(z - z_0)]\Big|_{z_0}^{z}$$

Thus, the atmospheric pressure variation in the troposphere is

$$p = p_0\left[\frac{T_0 - \alpha(z - z_0)}{T_0}\right]^{g/\alpha R} = p_0\left[\frac{T}{T_0}\right]^{g/\alpha R} \tag{3.18}$$

Example 3.7 shows how to apply Eq. (3.18) to find pressure at a specified elevation in the troposphere.

$$\ln p - \ln p_0 = \frac{g}{\alpha R}(\ln T - \ln T_0) \Rightarrow \ln\left(\frac{p}{p_0}\right) = \frac{g}{\alpha R}\ln\left(\frac{T}{T_0}\right) = \ln\left[\left(\frac{T}{T_0}\right)^{g/\alpha R}\right]$$

since $e^{\ln x} = x$

Pressure Variation in the Lower Stratosphere

In the lower part of the stratosphere (13.7 to ~~16.8 km above~~ the earth's surface as shown in Fig. 3.9), the temperature is approximately constant. In this region, integration of Eq. (3.17) gives

$$\ln p = \frac{zg}{RT} + C$$

At $z = z_0$, $p = p_0$, so the preceding equation reduces to

$$\frac{p}{p_0} = e^{-(z-z_0)g/RT}$$

so the atmospheric pressure variation in the stratosphere takes the form

$$p = p_0 e^{-(z-z_0)g/RT} \tag{3.19}$$

where p_0 is pressure at the interface between the troposphere and stratosphere, z_0 is the elevation of the interface, and T is the temperature of the stratosphere. Example 3.5 shows how to apply Eq. (3.19) to find pressure at a specified elevation in the troposphere.

$1\,J = N\,m$

$1\,N = 1\,kg \cdot \frac{m}{s^2}$

$$\frac{\frac{m}{s^2}}{\frac{°K}{m} \cdot \frac{J}{kg\,°K}}$$

$\frac{m^2}{s^2} \cdot \frac{kg}{N\,m}$ = no units

unitless constant

EXAMPLE 3.4

Predicting Pressure in the Troposphere

Problem Statement

If the sea level pressure and temperature are 101.3 kPa and 23°C, what is the pressure at an elevation of 2000 m, assuming that standard atmospheric conditions prevail?

Situation

Standard atmospheric conditions prevail at an elevation of 2000 m.

Goal

p(kPa absolute) ⬅ atmospheric pressure at $z = 2000$ m

Plan

Calculate pressure using Eq. (3.18).

Action

$$p = p_0\left[\frac{T_0 - \alpha(z - z_0)}{T_0}\right]^{g/\alpha R}$$

where $p_0 = 101{,}300$ N/m², $T_0 = 273 + 23 = 296$ K, $\alpha = 5.87 \times 10^{-3}$ K/m, $z - z_0 = 2000$ m, and $g/\alpha R = 5.823$. Then

$$p = 101.3\left(\frac{296 - 5.87 \times 10^{-3} \times 2000}{296}\right)^{5.823}$$

$$= \boxed{80.0 \text{ kPa absolute}}$$

$g = 9.81 \frac{m}{s^2}$

$R_{air} = 287 \frac{J}{kg\,°K}$

$\frac{g}{\alpha R} = \frac{9.81}{5.87 \times 10^{-3} \times 287} = \underline{5.823}$

EXAMPLE 3.5

Calculating Pressure in the Lower Stratosphere

Problem Statement

If the pressure and temperature are 2.31 psia ($p = 15.9$ kPa absolute) and −71.5°F (−57.5°C) at an elevation of 45,000 ft (13.72 km), what is the pressure at 55,000 ft (16.77 km), assuming isothermal conditions over this range of elevation?

Situation

Standard atmospheric conditions prevail at an elevation of 55,000 ft (16.77 km).

Goal

p ⬅ Atmospheric pressure (psia and kPa absolute) at an elevation of 55,000 ft (16.77 km)

Plan

Calculate pressure using Eq. (3.19).

Action

For isothermal conditions,

$$T = -71.5 + 460 = 388.5°R$$

$$p = p_0 e^{-(z-z_0)g/RT} = 2.31e^{-(10{,}000)(32.2)/(1716 \times 388.5)}$$

$$= 2.31e^{-0.483}$$

Therefore the pressure at 55,000 ft is

$$\boxed{p = 1.43 \text{ psia}}$$

SI units

$$\boxed{p = 9.83 \text{ kPa absolute}}$$