Evolution of Internal Waves in Hydrostatic Models

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LONG-TERM GOALS

My long-term goal is improving our understanding of interactions between subgrid-scale models and the numerical errors affecting simulation skill. My emphasis is on examining the effects of the necessary modeling compromises at practical grid resolutions for coastal and ocean modeling. For prognostic simulations, slow accumulation of truncation-level and subgrid-scale model errors will progressively distort simulation reliability. Understanding such errors and modeling the neglected subgrid-scale processes are necessary to 1) improve the predictive skill of models, 2) identify the temporal horizon over which a model has validity

OBJECTIVES

This project examines the un-physical behavior of internal waves when modeled under the hydrostatic approximation (commonly used in large-scale predictive models). The motivation is to develop improved modeling techniques that account for non-hydrostatic transfer of energy from resolved large-scale internal waves into subgrid-scale internal waves. These higher frequency waves can break on sloping boundaries or lead to enhanced wave-wave interaction, thereby amplifying vertical mixing and changing the spatial and temporal evolution of the oceanic density structure. Milestones towards this objective are: 1) quantifying the error accumulation in hydrostatic models and 2) developing methods to identify the localized time and space onset of non-hydrostatic behaviors in a hydrostatic model.

APPROACH

My graduate students and I are investigating the modeled behavior of internal waves by quantitatively comparing hydrostatic and non-hydrostatic models for a range of internal waves. Nonlinear terms in the momentum equations cause an initially-linear internal wave to steepen, resulting in nonlinear and (what should be) non-hydrostatic evolution. However, in a hydrostatic model, the non-hydrostatic dispersion is missing; therefore modeled internal waves steepen until numerical diffusion, dispersion, or dissipation provides balance. Thus, with the onset of non-hydrostatic physical behavior, a hydrostatic model is solving the wrong governing equations. It follows that improving model grid resolution in a hydrostatic model cannot provide a better solution in such locations since you are getting a better solution to the wrong set of equations.

We are examining the divergence of results between the hydrostatic and non-hydrostatic models under varying model conditions (e.g. grid resolution, time step, stratification). By separately quantifying errors in diffusion of mass and momentum, we have found a means of predicting required model parameters for adequately resolving internal waves with a hydrostatic code and a localized non-hydrostatic solution. Ms. Sarah Kelly Delavan (MS 2003) has studied how the error accumulates in hydrostatic models in 3 different forms: numerical dissipation of energy, numerical diffusion of mass, and numerical dispersion of waves (Hodges and Delavan 2004; Delavan 2003). Ms. Bridget Wadzuk (MS 2002, PhD expected 2004) has studied how to predict the onset of non-hydrostatic behavior as a means of *a priori* isolating problem regions in a hydrostatic model.

WORK COMPLETED

This project has supported two MS degrees (Wadzuk 2002, Delavan 2003) and a PhD student (Wadzuk) who will finish in Fall 2004. The past year has seen a convergence of Ms. Delavan's work (quantifying the accumulation of numerical errors in hydrostatic models of internal waves) with Ms. Waduzk's work (identifying the onset of non-hydrostatic behavior and comparing hydrostatic and non-hydrostatic behavior). Several distinct efforts have been completed during this project. We have developed a method for compensating for numerical mass diffusion errors (Laval et al. 2003a,b). We have developed a bottom-boundary layer model that reduces errors associated with density underflows (Dallimore et al. 2003, 2004). We have developed theory and computer code for analyzing the temporal evolution of numerical dissipation and numerical diffusion (Delavan and Hodges 2003; Hodges and Delavan 2004). We have added a non-hydrostatic pressure solution as a add-on to a hydrostatic model (Wadzuk and Hodges 2003). We have developed and tested a theory for isolating where and when non-hydrostatic effects become important in a hydrostatic model. This latter effort has been the principle focus of the past year. Regions where non-hydrostatic pressure contributes to internal wave evolution (e.g. at steep wave fronts) have been isolated by comparing hydrostatic and non-hydrostatic solutions. Examination of model results led to development of a theory for approximating the non-hydrostatic pressure effects in a hydrostatic model. The approximate non-hydrostatic pressure effects are scaled by horizontal acceleration to predict regions where the hydrostatic assumption is beginning to fail (Wadzuk and Hodges, 2004).

RESULTS

We have learned (Hodges and Delavan, 2004) that a finer model grid resolution may increase the rate of error accumulation in a hydrostatic model of an internal wave. This result is entirely counter to our fundamental notion of model convergence with refinement in grid size, and so requires some explanation. Models of internal waves have three principle forms of error: 1) numerical dissipation of energy, 2) numerical diffusion of mass, and 3) numerical dispersion of waves. These errors can be quantified in a model of an inviscid wave in a diffusionless medium by examining, respectively: 1) dissipation of kinetic and available potential energy – i.e. the "dynamic energy" or E_D; 2) increase in the background potential energy, E_B – i.e. the artificial diapycnal fluxes of mass that cause heavier fluid to be mixed upward in the water column; and 3) the change in the wavelengths in the power spectral density, PSD, of the pycnocline displacement. Figure 1 shows a dramatic change in the character of the E_D dissipation and E_B accumulation at about 7 wave periods. In Figure 2, the PSD is shown to shift to smaller wavelengths at this same time. The change in characteristics at 7 wave periods occurs due to the artificial steepening of the hydrostatic wave. When the wave steepens to the point that non-hydrostatic effects should be considered, the hydrostatic model disperses wave energy to smaller wavelengths (Figure 2), which allows more rapid numerical dissipation of energy (Figure 1a) and more rapid numerical diffusion of mass (Figure 1b). Because a finer horizontal resolution will support smaller wavelengths, the finer grid is able to disperse into smaller wavelengths than the coarser grid, which results in greater error accumulation for the finer grid.

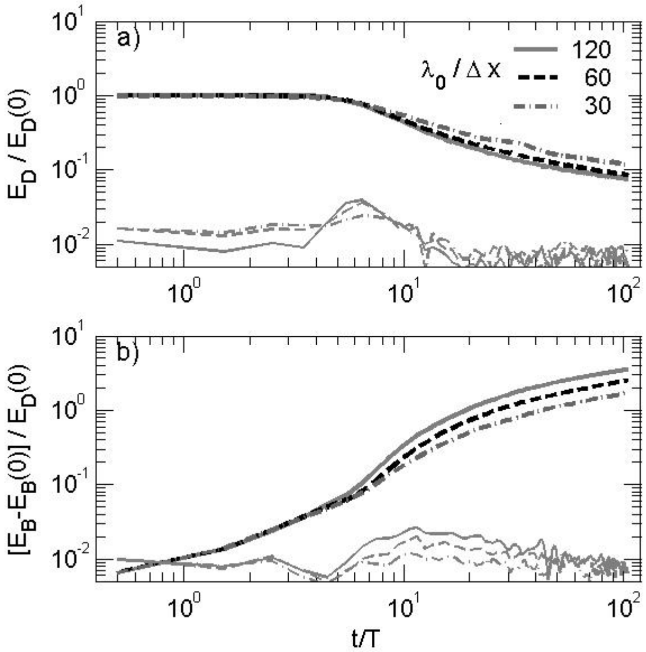

Figure 1. Line graphs showing effects of refining horizontal grid resolution for hydrostatic model of an internal wave. Lines show time evolution of energy for different grid resolutions. Upper panel (a) shows dynamic energy, E_D, change over time. Three lines for different grid resolutions remain close to the initial value, $E_D(0)$, for approximately 7 wave periods, then rapidly dissipate until only 10% remains at 100 wave periods. The dynamic energy dissipation rate increases as the resolution is improved from 30, to 60 to 120 grid cells per wavelength. The lower panel (b) shows the background potential energy, E_B, slowly increases to about 110% of the initial dynamic energy in 7 wave periods for all grid resolutions; subsequent rapid accumulation of error leads to the total background potential energy being greater than the initial dynamic energy at approximately 20 wave periods. The rate of background potential energy accumulation above 7 wave periods is greater as the grid resolution is improved. Both graphs show confidence intervals for binning of data over each wave period is of order 10^{-2}

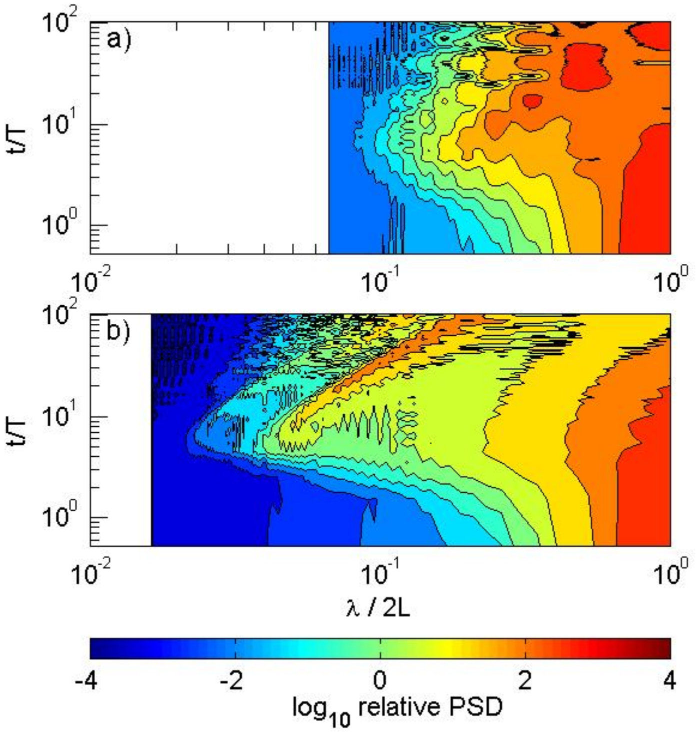

Figure 2. Contour graphs of power spectral density (PSD) for internal wave displacement at coarse (upper) and fine (lower) grid resolution. At coarse resolution, the power is at λ_0 until 7 T, when the power shifts to λ that is approximately 20% of λ_0. The power moves back to the larger λ as energy is dissipated between 7 and 100 T. By 100 T the power spike is back at λ_0. The fine grid results, lower panel, are similar, except that the power is transferred to a λ that is about 5% of λ_0 and by 100 T the principle power spike is located at only 20% of λ_0.

While the above results have quantified how hydrostatic error evolves, in Waduzk and Hodges 2004 we demonstrated that we can predict the time and place where non-hydrostatic pressure becomes important *using only a hydrostatic model*. This is a critical new ability that will provide a basis for improving existing hydrostatic models. In effect, we are demonstrating that we can "pull ourselves up by our bootstraps" – using only hydrostatic model results to find the places where the hydrostatic approximation is beginning to break down. We use a non-hydrostatic parameter (γ) defined as

$$\gamma_{nh} \equiv \frac{\left(-\frac{1}{\rho_o}\frac{\partial P_{nh}}{\partial x}\right)}{\left(\left.\frac{DU_{nh}}{Dt}\right|_{max,n} - \frac{DU_{nh}}{Dt}\right)} \tag{1}$$

$$\gamma_{h} \equiv \frac{\left(\frac{\partial}{\partial x}\int \frac{DW_h}{Dt}\,dz\right)}{\left(\left.\frac{DU_{h}}{Dt}\right|_{max,n} - \frac{DU_{h}}{Dt}\right)} \tag{2}$$

where P_{nh} is the non-hydrostatic pressure, W_h, is the vertical hydrostatic velocity, U is the horizontal velocity, and D/Dt indicates a Lagrangian derivative. Eq (1) is the parameter computed in a non-hydrostatic model, while eq. (2) is the approximation in a hydrostatic model. Figure 3 shows a comparison of γ_{nh} and γ_h. The correspondence show we can determine where non-hydrostatic processes are becoming important within a hydrostatic model.

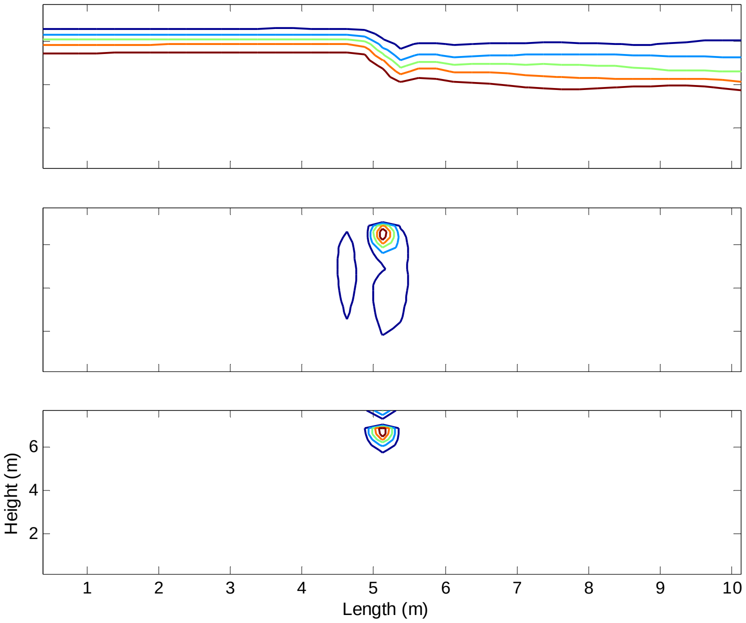

Figure 3: Graphs illustrating non-hydrostatic isolation. Upper frame shows isopycnals of an internal wave with a steep front. Middle frame shows contours of the approximate non-hydrostatic parameter based on a hydrostatic solution. Lower frame shows contours of the non-hydrostatic parameter computed from a non-hydrostatic solution. Comparison shows that the approximation from the hydrostatic model is a good predictor of size and position of the non-hydrostatic effects.

IMPACT/APPLICATIONS

The present research is likely to have a significant impact on near-future model development for the Navy. There is a need to develop approximation methods for non-hydrostatic effects that can be applied in the framework of hydrostatic models, as it is unlikely that full non-hydrostatic models will be practical for the Navy's ocean and coastal ocean prediction programs in the near future. The present research has shown that it may be feasible to use a hydrostatic model to predict the time/space locations where the model approximations are failing. With knowledge of the isolated regions of non-hydrostatic behavior, it may be possible to develop a localized non-hydrostatic solution or an adaptive mesh strategy that will improve the model skill without requiring a non-hydrostatic solution over the entire domain. As a side note, my PhD student, Bridget Wadzuk, presented her research under this project in an interview at Villanova University and will be a new assistant professor in spring 2005.

RELATED PROJECTS

None.

PUBLICATIONS

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