Evolution of Internal Waves in Hydrostatic Models

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LONG-TERM GOALS

My long-term goal is improving our understanding of interactions between sub-grid scale models and the numerical errors affecting simulation skill. For prognostic simulations, slow accumulation of truncation-level errors will progressively distort model results. Present sub-grid scale models selectively address certain processes (e.g. turbulence) while neglecting other processes (e.g. small-scale internal waves, particulate nature of algae). The small-scale nonlinear errors can accumulate over time, so understanding such errors and modeling the neglected processes are necessary to improve the predictive skill of models.

OBJECTIVES

I am examining the un-physical behavior of internal waves modeled under the hydrostatic approximation, which is commonly used in large-scale predictive models. The objective is to develop improved modeling techniques that account for transfer of energy from resolved internal waves into sub-grid scale internal waves. These higher frequency waves can break on sloping boundaries or lead to enhanced wave-wave interaction, thereby amplifying vertical mixing and changing the spatial and temporal evolution of the oceanic density structure.

APPROACH

To develop better modeling of internal waves we are presently focused on quantifying the error behavior of existing models. Nonlinear terms in the momentum equations slowly cause an initially-linear internal wave to steepen, resulting in nonlinear and what should be non-hydrostatic evolution. However, in a hydrostatic model, the non-hydrostatic dispersion is missing; therefore modeled internal waves steepen until numerical diffusion, dispersion, or dissipation provides balance. Thus, steepening internal waves in a hydrostatic model will artificially diffuse the density gradients, disperse the waves or dissipate their energy. Alternatively, a theoretically error-free hydrostatic model must result in waves that always steepen until breaking – thus artificially diffusing the density gradient in breaking-induced mixing. We are examining approaches for modeling the correct steepening/dispersion balance without resorting to the computational expense of a full non-hydrostatic solution over the entire model domain. I am working with two University of Texas (UT) graduate students, Ms. B. Wadzuk and Ms. S. Delevan, applying hydrostatic and non-hydrostatic models of the Navier-Stokes equations to simulate internal waves in simple geometries. These results are being compared to laboratory experiments and the KdV wave model of Horn et al 2000. We are examining the divergence of results between the hydrostatic and non-hydrostatic models under varying model conditions (e.g. grid resolution, time step, stratification). By separately quantifying errors in diffusion of mass and momentum, we are looking for a means of predicting required model parameters for adequately resolving internal waves with a hydrostatic code. The second year of this project has focused on three issues: 1) analysis of numerical diffusion and dissipation of evolving internal waves, 2) developing concepts for a "localized" non-hydrostatic pressure solution within an existing hydrostatic numerical model, and 3) analysis of the numerical accuracy of the common 2nd order semi-implicit approach for the discrete Navier-Stokes equations. The first issue has been addressed in the M.S. thesis of Ms. Sarah Kelly Delavan, who graduated in August 2003. The second issue is the ongoing work of Ms. Bridget Wadzuk, whom has just completed her first year as a Ph.D. student. The third issue resulted in a paper that was submitted to the Journal of Engineering Mechanics.

WORK COMPLETED

My graduate student, Ms. Delavan completed her thesis on *Accumulation of Numerical Errors in Hydrostatic Models of Internal Waves*. Her work builds on last year's work with Prof. Laval at UBC (Laval et al. 2003a, 2003b). Our work has shown that, in contrast to conventional expectation, the numerical dissipation and diffusion errors for hydrostatic models of internal waves do not always decrease with finer grid resolution. We conducted 60 numerical simulations over a variety of stratifications, internal wave characteristics, and grid sizes. We developed a set of analysis codes to compute the temporal evolution of numerical dissipation and numerical diffusion for each simulation. A poster was presented at the ASLO spring meeting (Delavan and Hodges, 2003) that showed the "Limitations of the shallow water equations for modeling stratified water bodies."

The ongoing Ph.D. work of Ms. Wadzuk has developed the basis for a "localized" non-hydrostatic model. She has developed a non-hydrostatic model built on top of an existing hydrostatic model so that the non-hydrostatic contribution to the pressure is solved as a small change from the hydrostatic pressure. The principle scientific advance of the past year is developing the concept for solving the non-hydrostatic pressure only in those areas where it is *both* important and resolvable. The non-hydrostatic pressure is generally significant only in isolated regions (e.g. near the front of an evolving internal wave or along littoral boundaries), and the ability to adequately resolve the non-hydrostatic pressure is fundamentally dependent on the model grid scale. Thus, the global solution of the non-hydrostatic pressure is generally unwarranted and is the principal driver of the large computational cost of a non-hydrostatic model. We have developed a concept for solving the non-hydrostatic pressure only where the combination of grid scale and hydrostatic velocities allow a significant effect. This concept is presently being developed and tested in the model. Ms. Wadzuk presented her work at the ASCE Engineering Mechanics conference this year (Wadzuk and Hodges, 2003).

In analysis of model accuracy during the above studies, it became apparent that the behavior of the Crank-Nicolson semi-implicit method as the model time step is refined had some peculiarities not previously noted in the literature. I completed an analysis of the derivation of the C-N semi-implicit approach for the hydrostatic equations and revealed an error in all prior works (e.g. Casulli and Cattani, 1994) that results in the 2nd order method being degraded to 1st order accuracy. I developed a new second-order correction method (Hodges, 2003a) and completed a detailed analysis of the error characteristics of the Crank-Nicolson method with 126 simulations (Hodges, 2003b).

RESULTS

We have shown that neglecting the non-hydrostatic pressure results in interplay between two forms of numerical error (diffusion and dissipation) that leads to non-monotonic error behavior as a model grid is refined. In effect, a finer grid scale may not always lead to a decrease in error: a decrease in numerical dissipation can cause an increase in numerical diffusion (due to wave steepening, Figure 1), which appears as excessive mixing and a poor representation of thermocline evolution. Refining the model grid at coarse resolutions provides the commonly-expected error decrease; however, the simulations reached a limiting value, after which further grid refinements cause the error to increase. This inconsistency appears fundamental to the solution of non-hydrostatic internal wave evolution in a hydrostatic code. The results can be interpreted as a transition in the dominant error term from numerical dissipation to numerical diffusion. Under a dissipation-dominated system, a high-order flux-limiting transport scheme can effectively control numerical diffusion, and the principal dissipative error causes damping of internal wave motions (limiting the wave steepness) while preserving the thermocline structure. Under a diffusion-dominated system, internal waves are allowed to steepen such that numerical diffusion is enhanced – even with the use of flux-limiting high-order transport; as a result, the predicted thermocline structure is eroded and the wave evolution may be altered.

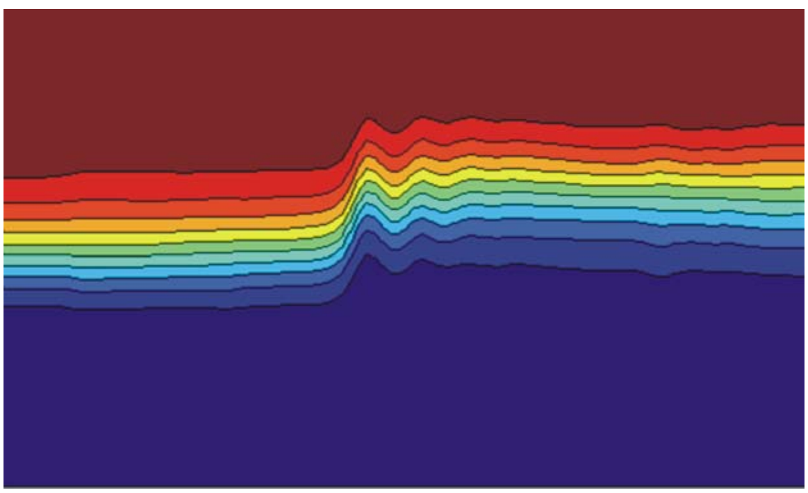

Figure 1. Steepening of an internal wave on a thermocline in a hydrostatic model. The characteristic wave shape is that of a propagating bore with a train of solitons, which is physically a non-hydrostatic phenomenon. This occurs serendipitously in a hydrostatic model of a nonlinear wave when the numerical dissipation balances nonlinear steepening. The resulting sharp front enhances numerical diffusion. Further decreases in numerical dissipation caused by refining the grid scale will result in further steepening of the wave front and increased numerical diffusion. Thus, refinement of the grid for the hydrostatic model can lead to increased error in the predicted thermal structure.

Our detailed study of the 2nd order Crank-Nicolson discretization (a common approach in semi-implicit coastal and ocean models) showed the method is typically only 1st order accurate in the common implementation. A new correction term was derived to provide 2nd order accuracy. This new method was shown to provide 2nd order accuracy under conditions where the common approach fails (Hodges, 2003b). However, it was also determined that when the CFL for the barotropic mode is greater than unity, any form of the C-N method was effectively only 1st order accurate. We have determined that this occurs because a high barotropic CFL effectively loses the linkage between the barotropic mode and the advection term.

IMPACT/APPLICATIONS

Our work has described the error behavior of a moving thermocline as the model grid is refined. The results show non-monotonic error behavior that impacts how we assess the predictive capabilities of present hydrostatic models. The prevailing assumption is that any "good" model must get "better" with a finer grid. Indeed, this is the reasoning for using nested fine-scale grids in many models. However, with our new understanding, it is clear that modelers need a better approach to quantifying the change in the error as a model moves from dissipation-dominated to diffusion-dominated regimes. In effect, when using the hydrostatic equations, a finer grid resolution provides a "better" solution to the wrong set of equations – i.e. at finer resolutions more of the non-hydrostatic behavior can be resolved, so the error due to neglect of the non-hydrostatic terms becomes dominant. This results in a shift from a dissipation-dominant to a diffusion-dominant error. Knowledge of the dominant error term at a particular grid scale should allow improved methods for data assimilation. Under a dissipation-dominated system, data assimilation should focus on sharpening gradients in the velocity field, whereas a diffusion-dominated system should have data assimilation that focuses on sharpening gradients in the density field. Thus, by providing a greater understanding of the characteristics of numerical errors, we provide the basis for improving our predictive capabilities.

The present study of the Crank-Nicolson discretization method brings forward a question on whether or not semi-implicit models should be used. Presently, the semi-implicit approach is often favored over an explicit approach to allow a stable solution at a larger model time step without requiring mode splitting. However, from our recent work it is clear that the accuracy of the semi-implicit approach is one order less than theory when a large time step is used. The only condition under which the semi-implicit method retains its formal order of accuracy is when the time step is small enough that an explicit method could be applied. It is expected that the submitted paper on this issue may be controversial as there is a significant body of prior research (including my own work) invested in semi-implicit methods.

TRANSITIONS

The background potential energy algorithm and density-filtering algorithm of Laval et al. (2003) is implemented in distribution version 1.5.2 of the CWR-ELCOM model (Hodges, et al. 2000). In addition to the present research program, the model is being used at Stanford University, University of Stuttgart, University of Western Australia, University of British Columbia, Instituto Nacional Del Agua Y Del Ambiente (Argentina), Kinneret Limnological Laboratory (Israel), University of Ioannina (Greece).

RELATED PROJECTS

The long-term improvement of models for sub-grid scale effects is the subject of ongoing collaborative efforts of the PI with Prof. Jörg Imberger (Stockholm Water Prize Laureate, 1997) at University of Western Australia. These efforts include development of a benthic boundary layer underflow model and development of an improved particle tracking algorithm. The benthic boundary layer model has had significant cross-over to the development of improved modeling of internal waves.

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