Evolution of Internal Waves in Hydrostatic Models

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LONG-TERM GOALS

My long-term goal is improving understanding of interactions between sub-grid scale models and numerical errors, which affect simulation skill for physical and biological processes in estuaries and coastal ocean environments. For prognostic simulations, slow accumulation of truncation-level errors progressively distorts model results. Present sub-grid scale models selectively address certain processes (e.g. turbulence) while neglecting other processes (e.g. small-scale internal waves, particulate nature of algae). Understanding the errors and modeling neglected processes are critical to improving prognostic skill when small-scale nonlinear errors can accumulate over time.

OBJECTIVES

I am examining the un-physical behavior of internal waves modeled under the hydrostatic approximation. This knowledge is a precursor to developing improved modeling techniques, accounting for transfer of energy from resolved internal waves into sub-grid scale internal waves. These higher frequency waves can break on sloping boundaries and enhance vertical mixing, thereby changing the temporal evolution of the density structure.

APPROACH

I am working with a graduate student using hydrostatic and non-hydrostatic 3D models of the Navier-Stokes equations to simulate internal waves in simple geometries. These results will be compared to laboratory experiments and the KdV wave model of Horn et al 2000. After validating the models, we will examine the divergence of results between the hydrostatic and non-hydrostatic models under varying model conditions (e.g. grid resolution, time step, stratification). By separately quantifying errors in diffusion of mass and momentum, we will look for a means of predicting required model parameters for adequately resolving internal waves with a hydrostatic code. Nonlinear terms in the momentum equations slowly cause a hydrostatic linear wave to steepen, resulting in nonlinear and, what should be, non-hydrostatic evolution. We will look for a means of modeling the correct steepening/dispersion balance without resorting to a full non-hydrostatic solution. Our first step for the upcoming year will be to add the dynamic pressure terms to an existing hydrostatic numerical model (ELCOM, Hodges et al. 2000). This will allow us to directly isolate the effects of the hydrostatic approximation while keeping all the other numerical model errors constant.

WORK COMPLETED

In the first three months of this project, I have been examining issues associated with our ability to control numerical diffusion errors when propagating density waves through a fixed grid in a hydrostatic code. This work is being done in collaboration with Prof. Jörg Imberger and a Ph.D. student, Bernard Laval, at the Centre for Water Research (CWR), University of Western Australia. Prof. Imberger invited me to visit for 4 weeks in July/August 2000 to examine performance of the hydrostatic code on these issues and complete a draft of a paper for submission. I also attended the 6th International Workshop on Physical Processes in Natural Waters in Girona, Spain. This small workshop provided the opportunity to meet with E. Bäuerle (University of Constance), J. Appt (University of Stuttgart), and A. Wuest (EAWAG, Switzerland) to discuss recent progress in modeling and collection of field data for internal waves. A primary goal in the first three months of this project was recruiting a graduate student to assist in developing new modeling approaches and performing data analyses of the results. I was successful in recruiting Ms. Bridget Wadzuk, a second-year graduate student at UT who has decided to continue on with studies for a PhD. after finishing her M.S. degree.

RESULTS

In hydrostatic models, the horizontal grid resolution has a significant effect on the evolution of modeled internal waves. Starting with an initial linear seiche (Figure 1a) a coarse grid resolution (Figure 1b) results in a linear seiching motion damped by numerical diffusion of mass and numerical dissipation of energy. As the grid is refined, both error effects are initially reduced, with the result that nonlinear terms (previously damped by numerical dissipation) cause the linear wave to steepen over time (Figure 1c). The magnitude of the steepening is incorrect, as it would be balanced by non-hydrostatic dispersion in the physical world. In a hydrostatic model, the steepening is balanced by numerical dissipation, which is affected by increased numerical diffusion of mass associated with the steep pycnocline angle relative to the fixed grid. An interesting observation is that the model skill in representing the physical world is not necessarily improved by grid refinement in a hydrostatic model. In the limit of ultimate grid refinement where numerical dissipation and diffusion are reduced to zero, a hydrostatic model does not have a physical process to balance nonlinear steepening, so an internal wave must, necessarily, steepen until it breaks.

Quantifying numerical diffusion associated with propagating a steep front is a prerequisite to improving our ability to model internal waves. In a paper submitted to the Journal of Hydraulic Engineering, we demonstrate that even small errors associated with numerical diffusion are detrimental to model skill due to their slow accumulation over the course of a simulation. We further demonstrate that numerical diffusion can be exactly computed on a system-wide basis using the concept of background potential energy. We propose an ad hoc method for counteracting diffusion by using a vertical filter to sharpen a pycnocline. While the results are promising, the method suffers significant drawbacks in 1) required computational time, 2) the global nature of the quantification of numerical diffusion, and 3) the corresponding global distribution of the sharpening correction.

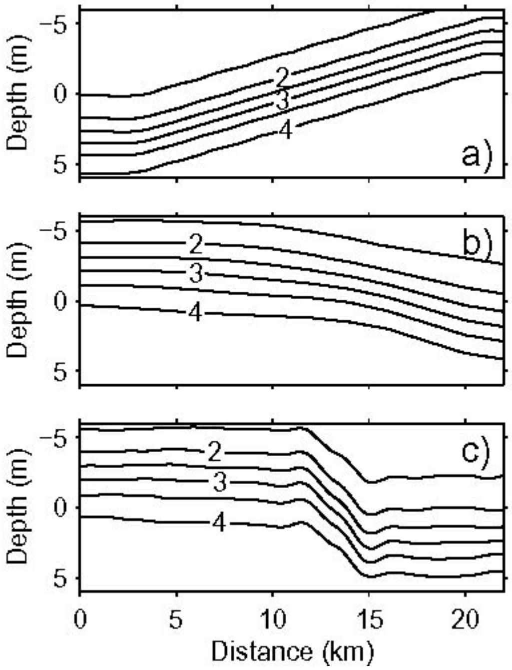

Figure 1. Density isopycnals of modeled internal waves in a hydrostatic simulations after eight days from a) initial condition using b) coarse grid of 2000m, or c) fine grid of 200 m.

IMPACT/APPLICATIONS

Improvements in our ability to correctly represent the evolution of internal waves in hydrostatic models will provide models with greater skill in representing the spatial and temporal resolution of the density field in the oceans. This will improve our ability to use model results to predict thermocline position and thickness, especially in coastal regions where steepening and breaking of internal waves along the slope may be an important source of mixing.

TRANSITIONS

None

RELATED PROJECTS

I presently have several unfunded collaborative efforts with the Centre for Water Research at the University of Western Australia, aimed at improving various aspects of the ELCOM hydrostatic numerical model. These include development of a benthic boundary layer underflow model and new approach for local sub-time stepping in a conservative transport scheme.

REFERENCES

Horn, D.A., L.G. Redekopp, J. Imberger and G.N. Ivey (2000), Internal wave evolution in a space-time varying field, *J. Fluid Mech.* **424**:279-301.

Hodges, B.R., J. Imberger, A. Saggio, and K. Winters, (2000), Modeling basin-scale internal waves in a stratified lake, *Limnol. Oceanogr.* **45**:1603-1620.

PUBLICATIONS

B. Laval, B.R. Hodges and J. Imberger, Numerical diffusion in 3D, hydrostatic Z-level lake models, submitted to *J. Hydr Engrg*, 2001.