

1 **A general framework for modeling shared**  
2 **autonomous vehicles**

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27 **Abstract**

28 Shared autonomous vehicles (SAVs) could provide low-cost service to travelers and possibly  
29 replace the need for personal vehicles. Previous studies found that each SAV could service  
30 multiple travelers, thereby eliminating several personal vehicle trips. However, a major  
31 limitation of previous studies is the reliance on custom software packages with unrealistic  
32 congestion models, network structures, or travel demand. For effective comparisons with  
33 personal vehicle scenarios, a common traffic flow simulator is necessary.

34 This paper presents an event-based framework for implementing SAV behavior in existing  
35 traffic simulation models. We demonstrate this framework in a cell transmission model-based  
36 dynamic network loading simulator. We also study a heuristic approach for dynamic ride-  
37 sharing. We compared personal vehicles and SAV scenarios on the downtown Austin city  
38 network. Without dynamic ride-sharing, the additional empty repositioning trips made by  
39 SAVs increased congestion and travel times. However, dynamic ride-sharing resulted in

40 travel times comparable to those of personal vehicles because ride-sharing reduced vehicular  
41 demand. Overall, the results show that realistic traffic flow models should be used for  
42 studying SAVs, but with well-chosen SAV fleets and routing algorithms, SAVs could provide  
43 acceptable service to travelers.

## 44 1 Introduction

45 Autonomous vehicles could revolutionize transportation. Adaptive cruise control could in-  
46 crease road capacity [16, 26] and reservation-based intersection control [7, 8] could do the  
47 same for intersections [13, 22]. The focus of this paper is on integrating models of these  
48 traffic flow improvements with shared autonomous vehicle (SAV) behavior. SAVs are a fleet  
49 of autonomous SAVs that provide low-cost service to travelers, possibly replacing the need  
50 for personal vehicles. Previous studies [1, 10] assuming that all travelers used SAVs found  
51 that each SAV could service multiple travelers, reducing the number of vehicles needed in  
52 the SAV fleet. Although 100% SAV use is unlikely to occur in the near future, previous  
53 results suggest great potential benefits when 100% SAVs becomes viable. Strategies such  
54 as preemptive relocation of SAVs for expected demand [10] or dynamic ride-sharing [11] are  
55 additional options for improving service.

56 However, a major limitation of previous studies is that many relied on custom software  
57 packages with unspecified or unrealistic congestion models [1, 10, 11, 27] and/or grid networks  
58 [10, 11]. Although these were important studies for technology demonstration purposes, they  
59 lacked realistic flow models. Many studies even assumed that link travel times were constant.  
60 This limitation prevents prevent accurate predictions of the benefits of SAVs.

61 It is clear from a review of previous work that a method of integrating SAVs with real-  
62 istic congestion models is a common issue without an obvious solution. Moreover, because  
63 researchers and practitioners use a variety of traffic models, it is desirable for SAVs to be  
64 able to be integrated within their preferred flow model. We address this problem by de-  
65 veloping an event-based framework for adding SAVs to a general class of existing traffic  
66 simulators. To further justify this framework, we also present results from a calibrated city  
67 network demonstrating that not using realistic congestion models can greatly exaggerate the  
68 potential benefits of SAVs.

69 This framework admits a dynamic network loading model of SAVs using the well-established  
70 cell transmission model (CTM) [5, 6]. We compare SAVs using heuristics for vehicle routing  
71 and dynamic ride-sharing based on previous work [10, 11] against personal vehicle scenarios.  
72 (Heuristics are used because the vehicle-routing problem is NP-hard [28].) The framework  
73 allows us to study SAV behaviors using a more realistic congestion model.

74 The contributions of this paper are as follows:

- 75 1. We propose an event-based framework for implementing SAVs in existing traffic models.  
76 This can be adapted for macro-, meso-, or micro-scopic flow models. Our results show  
77 that SAVs can cause significant congestion, so using realistic traffic flow models is  
78 necessary for accurate estimations of SAV level of service. Therefore, future work on

79 SAVs should consider using this framework or others to incorporate realistic network  
80 models.

- 81 2. We demonstrate this framework by studying congestion when SAVs are used to service  
82 all travelers, using CTM to propagate flow. We also describe and study a heuristic  
83 for dynamic ride-sharing on the downtown Austin city network and compare it with  
84 personal vehicle results from dynamic traffic assignment (DTA).
- 85 3. We compare SAV scenarios (including dynamic ride-sharing), with personal vehicle  
86 scenarios on the calibrated downtown Austin city network. Overall, results show that  
87 a smaller SAV fleet can service all travel demand in the AM peak. However, some  
88 SAV scenarios also increased congestion because of the additional trips made to reach  
89 travelers' origins. Therefore, it is important to model congestion when studying SAVs  
90 to attain realistic estimates of quality of service. Furthermore, SAVs may be less  
91 effective than previously predicted for peak hour scenarios. Nevertheless, SAVs with  
92 dynamic ride-sharing provided service comparable to personal vehicles.

93 The remainder of this paper is organized as follows: Section 2 discusses recent develop-  
94 ments in AV traffic flow and SAV modeling. Section 3 describes a general framework for  
95 SAVs. In Section 4, we describe specific behaviors used in our case study. We present exper-  
96 imental results for SAVs and compare with personal vehicle scenarios in Section 5. Section  
97 6 presents our conclusions.

## 98 2 Literature review

99 SAVs differ from personal vehicles as follows:

- 100 • With personal vehicles, each traveler drives a vehicle from the origin to the destination,  
101 then is assumed to park at the destination. Travelers choose routes to minimize their  
102 own travel time, resulting in a dynamic user equilibrium (DUE) in which no vehicle  
103 can improve travel cost by changing routes.
- 104 • With SAVs, all travelers are serviced by SAVs, and no personal vehicles are used.  
105 When travel demand is ready to depart, an SAV drives to the origin, takes the traveler  
106 to the destination, and then becomes available to service other demand. This may  
107 result in some empty repositioning trips to reach travel demand, but the total number  
108 of vehicles on the road may be reduced.

109 Naturally, SAV behavior raises cost and security issues. SAVs are essentially a fleet of  
110 driverless taxis, and replacing personal vehicles with taxis is not cost-effective for most  
111 travelers. However, because SAVs are driverless, the cost of travel is much less and is more  
112 similar to the costs of vehicle ownership [12]. SAVs may also raise security concerns due to  
113 their vulnerability to hacking. However, security issues with SAV implementation are outside  
114 the scope of this paper. Complete replacement of personal vehicles by SAVs has been studied

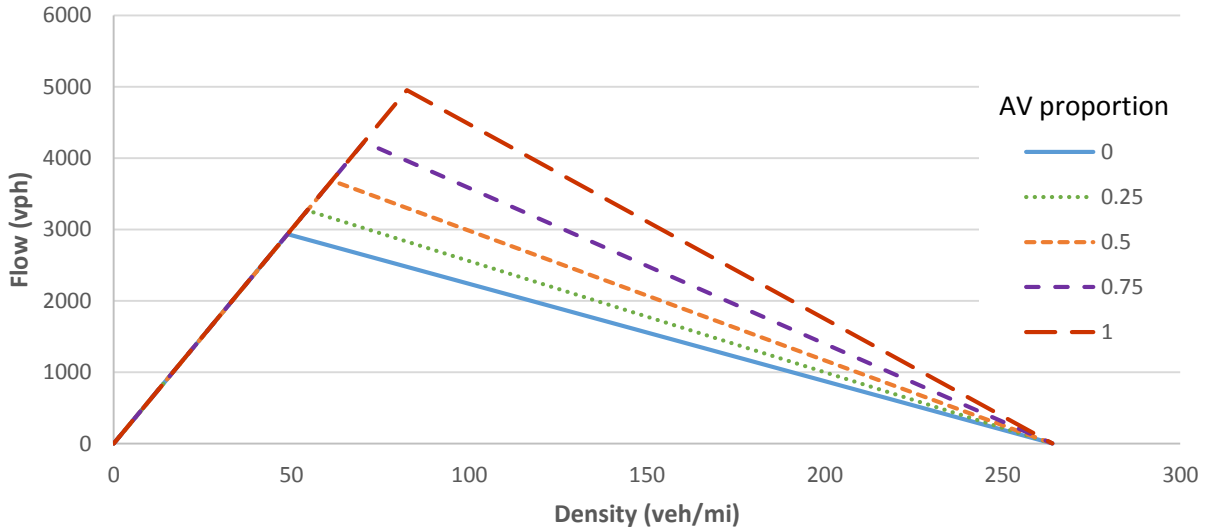


Figure 1: Flow-density relationship as a function of AV proportion for a free flow speed of 60 mph [19]

115 by previous work [11, 12], and the purpose of this paper is to improve the accuracy of such  
 116 models.

117 This paper builds on previous work on AV traffic flow and intersection control models  
 118 (Section 2.1) and SAVs (Section 2.2) to model SAV behavior.

## 119 2.1 Traffic models of autonomous vehicles

120 After years of development culminating in AV testing on public roads, the literature has  
 121 begun to focus on modeling new traffic behaviors available to AVs. Adaptive cruise control  
 122 could increase capacity [16, 26] and traffic flow stability [21, 25]. However, Levin & Boyles [17]  
 123 showed that increased road capacity may be offset by greater travel demand, particularly  
 124 for empty repositioning trips. Therefore, the flow-density relationship is likely to change in  
 125 space and time with the proportion of AVs. Levin & Boyles [19], developed a multiclass  
 126 hydrodynamic theory with varying flow-density relationship, and solved it using a multiclass  
 127 extension of the cell transmission model [5, 6]. Furthermore, they proposed a first-order  
 128 car-following model to predict the flow-density relationship as a function of the proportion  
 129 of AVs, with an example shown in Figure 1.

130 Dresner & Stone [7, 8] developed *reservation-based intersection control*: vehicles com-  
 131 municate wirelessly with an intersection manager to reserve a space-time path through the  
 132 intersection. The intersection manager simulates the path on a grid of tiles and accepts the  
 133 request only if it does not conflict with the reservations of other vehicles. Reservations make  
 134 greater use of intersection capacity, allowing reductions in delays beyond optimized traffic  
 135 signals in some scenarios [13, 22]. However, due to the computational complexity of the

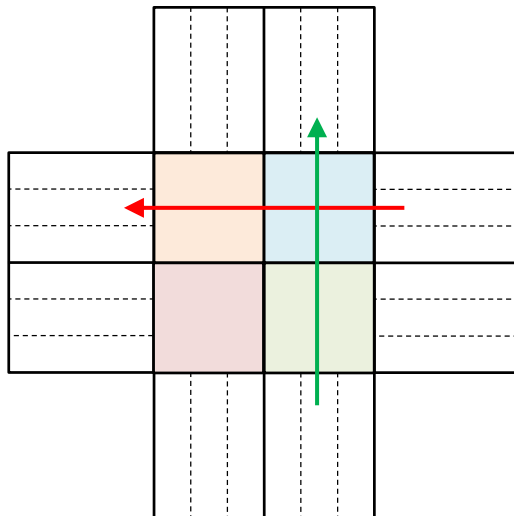


Figure 2: Conflict region model

136 reservation protocol, many previous studies have been limited to small networks [15] or used  
 137 simplified intersection models that reduced the traffic efficiency [2, 3]. Levin & Boyles [18]  
 138 developed the conflict region model of reservations, which is tractable for large-scale DTA,  
 139 and is therefore used in the simulations in this paper. Instead of simulating vehicle paths  
 140 along a fine grid of tiles, the conflict region model aggregates tiles into larger *conflict regions*.  
 141 The conflict regions for a four-way intersection are illustrated in Figure 2. Vehicle turning  
 142 movements are limited by the capacity of all conflict regions the vehicle must pass through.  
 143 Different turning movements pass through different sets of conflict regions; for example,  
 144 left-turning traffic passes through more conflict regions than right-turning traffic.

## 145 2.2 Shared autonomous vehicles

146 Multiple studies have investigated the possibility of using a fleet of SAVs to reduce reliance  
 147 on personal vehicles and improve mobility and safety [9]. Fagnant & Kockelman [10] es-  
 148 timated that one SAV could provide service to around eleven travelers on a grid network  
 149 approximation of Austin, Texas with most travelers waiting at most 5 minutes for pick-up,  
 150 although vehicle travel time increased. Fagnant & Kockelman [11] incorporated dynamic  
 151 ride-sharing, and found that it could offset the additional vehicle travel time. However, only  
 152 10% of personal trips of Austin were included. Further studies on different cities have sup-  
 153 ported indications that a smaller fleet of SAVs could provide service to all travelers. Burns  
 154 et al. [1] studied a centrally dispatched SAV system in three different urban and suburban  
 155 environments. Their findings indicated that a much smaller fleet of SAVs could provide ser-  
 156 vice to all residents with acceptable waiting times. Also, a slightly reduced fleet of taxicabs  
 157 could improve on wait times and vehicle utilization in Manhattan, New York. Spieser et  
 158 al. [27] found that a SAV fleet one-third the size of the personal vehicle fleet was sufficient  
 159 for providing service to Singapore travelers.

160 Although the results of previous studies are encouraging, they relied on unrealistic traffic  
161 congestion models, such as using fixed link travel times [10, 12, 27]. In addition, several  
162 studies used grid-based network approximations of cities [10]. SAVs could actually increase  
163 the number of trips, as well as vehicle miles traveled, by making repositioning trips to reach  
164 new travelers. These increases in demand could result in significantly higher congestion in  
165 saturated urban cities. Unfortunately, due to the lack of realistic congestion modeling, the  
166 traffic congestion and traveler service benefits of SAVs reported by previous studies may be  
167 greatly exaggerated. The lack of realistic congestion models across most previous studies  
168 indicates that the problem of integrating SAVs with established traffic flow models does not  
169 have an obvious solution. Therefore, this paper presents an event-based framework to build  
170 an SAV simulation on top of a general class of existing traffic simulators. We hope this will  
171 encourage future studies on SAVs to use more realistic congestion models to obtain more  
172 accurate predictions.

### 173 3 Shared autonomous vehicle framework

174 This section presents a general framework for dynamic simulation of SAVs to admit the  
175 latest developments in traffic flow modeling and SAV behavior. The framework is built on  
176 two events that can be integrated into most existing simulation-based traffic models. The  
177 purpose of this framework is to encourage future studies on SAVs to make use of existing  
178 traffic models for effective comparisons with current traffic conditions. As the case study will  
179 demonstrate, replacing personal vehicles with SAVs for the same number of travelers could  
180 increase congestion. To determine whether SAVs are beneficial, it is therefore necessary to  
181 compare SAV and personal vehicle scenarios in the same traffic model.

182 This section discusses the key events defining this framework and the types of responses  
183 they warrant. However, the specific responses depend on the dispatcher logic, and for gener-  
184 ality this framework does not require specific dispatcher behaviors. Section 4 discusses the  
185 dispatcher logic used in our case study, including dynamic ride-sharing.

186 This framework is based on a traffic simulator operating on a *network*  $G = (N, A, Z, V, D)$ ,  
187 where  $N$  is the set of nodes,  $A$  is the set of links, and  $Z \subset N$  is the set of centroids. The  
188 network has a set of SAVs  $V$  that provide service to the demand  $D$ . Note that  $D$  is in terms  
189 of person trips, not vehicle trips, since travelers will be serviced by SAVs. The integration of  
190 the framework with the traffic simulator is illustrated through the simulator logic in Figure  
191 3, with simulator time  $t$  and time step  $\Delta t$ . Events and responses are indicated with double  
192 lines; the remainder is the standard traffic simulator. The simulation steps are grouped into  
193 three modules: 1) demand; 2) SAV dispatcher; and 3) traffic flow simulator. The remainder  
194 of this section discusses these modules in greater detail.

#### 195 3.1 Demand

196 The demand module introduces demand into the simulation. At each time  $t$ , the demand  
197 module outputs the set of travelers that request a SAV at  $t$ . (This does not include waiting

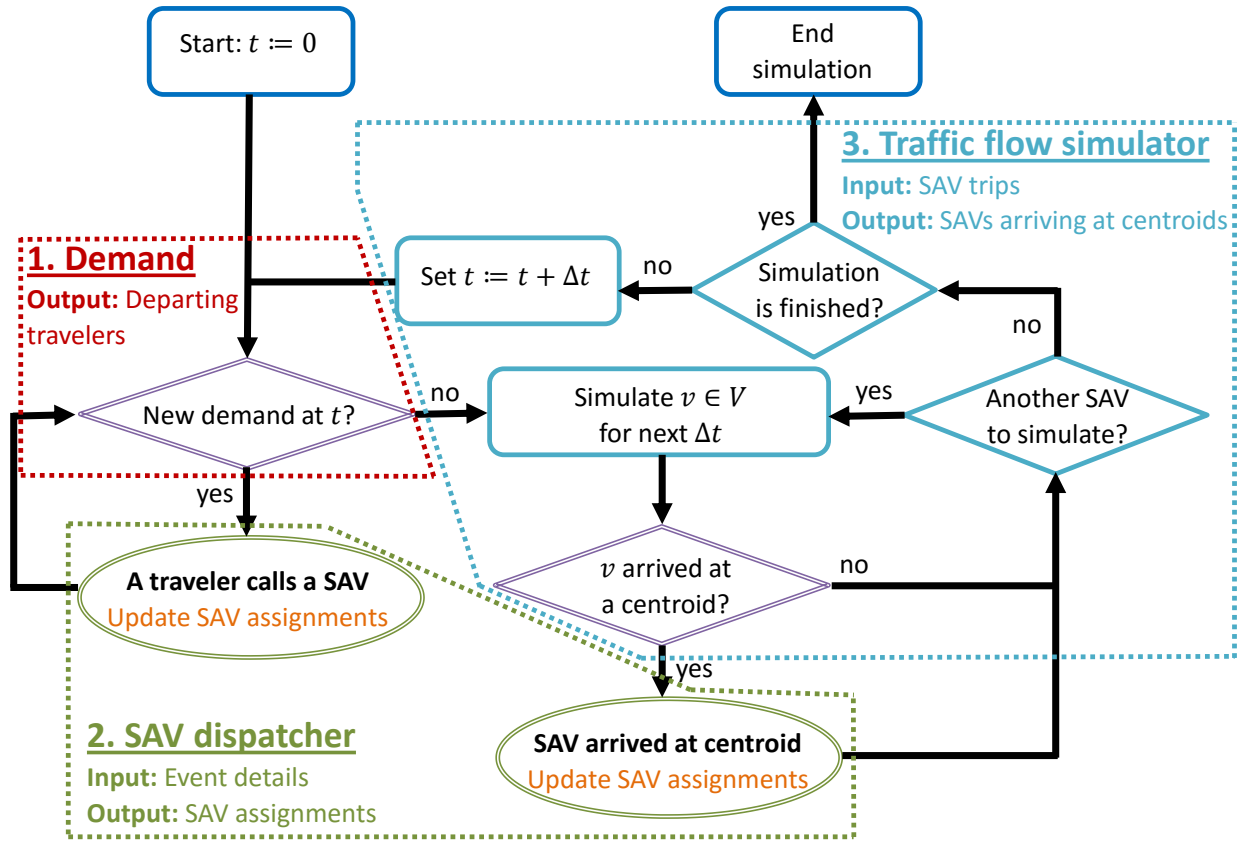


Figure 3: Event-based framework integrated into traffic simulator

198 travelers.) The demand module of existing traffic simulators may be adapted for this purpose,  
199 with the caveat that the demand is in the form of travelers, not personal vehicles. If new  
200 demand appears at  $t$ , this triggers the corresponding event: a traveler calls a SAV.

201 Because SAV actions are triggered by a traveler calling a SAV, this framework admits  
202 a very general class of demand models. The major requirement is that demand must be  
203 separated into packets that spawn at a specific time with a specific origin and destination.  
204 Although this paper primarily refers to demand as individual travelers, these packets could  
205 also represent a group of people traveling together. Demand cannot be continuous over  
206 time because that would trigger a very large number of events. However, in our case study  
207 demand and traffic flow are simulated at a timestep of 6 seconds, which is demonstrated to  
208 be computationally tractable for city networks.

209 As a result, this framework can handle both real-time and pre-simulation demand gener-  
210 ation. Real-time demand may be randomly generated every simulation step, triggering the  
211 event of a traveler calling a SAV when the demand is created. For models with dynamic  
212 demand tables, each packet of demand spawns at its departure time and calls a SAV then.  
213 In addition, if demand is assumed to be known prior to its departure time, SAVs may choose  
214 to preemptively relocate before the traveler appears. However, this requires that travelers  
215 plan ahead to schedule a SAV before they depart. A less restrictive assumption is that  
216 the productions at each zone are known, and SAVs may preemptively relocate in response  
217 to expected travelers. This requires less specific information about the traveler, and trip  
218 productions are usually predicted by metropolitan planning organizations.

## 219 3.2 SAV dispatcher

220 This framework assumes the existence of a central SAV dispatcher that knows the status  
221 of all SAVs and can make route and passenger assignments. With the range of wireless  
222 communication available today, the existence a central dispatcher is a reasonable assumption  
223 for SAVs. However, if desired the dispatcher logic could also be chosen to simulate SAVs  
224 making individual decisions on their limited information.

225 The SAV dispatcher module determines SAV behavior, including trip and route choice,  
226 parking, and passenger service assignments. The dispatcher operates as an *event handler*  
227 responding to the events of a traveler calling a SAV or a SAV arriving at a centroid, and  
228 takes as input the event details. The dispatcher is responsible for ensuring that all active  
229 travelers are provided with SAV service.

230 The output of the dispatcher are the SAV behaviors in response to the event. These  
231 include SAV vehicle trips (which are passed to the traffic flow simulator), passenger pick-up  
232 and drop-off, and parking SAVs that are not needed. At any given time, each SAV is either  
233 parked at a centroid or traveling. If a SAV is parked, its exact location must be known.

234 This framework is event-based, meaning that SAV actions are assigned when one of the  
235 following events occurs:

- 236 1. A traveler calls a SAV.
- 237 2. A SAV arrives at a centroid.



238 The first event is triggered in response to demand departing (or requesting to depart), and  
239 the second is in response to a SAV completing its assigned trip. These can be implemented  
240 in most simulation-based frameworks. Instead of a traveler departing by creating a personal  
241 vehicle, the traveler calls a SAV. When a SAV completes travel on a path (which should end  
242 in a centroid), this also triggers an event so the simulator can check for arriving or departing  
243 passengers at that centroid and assign the SAV on its next trip.

### 244 3.2.1 A traveler calls a SAV

245 When a traveler  $d \in D$  calls a SAV, the dispatcher should ensure that the demand will be  
246 satisfied by a SAV. This could occur in several ways:

- 247 1. If an empty SAV  $v \in V$  is parked at  $d$ 's origin, the dispatcher might assign  $v$  to  
248 immediately pick up  $d$ .
- 249 2. If an empty SAV  $v \in V$  is parked elsewhere, the dispatcher may assign  $v$  to travel to  
250  $d$ 's origin. In this case, the dispatcher might choose to wait to optimize the movement  
251 of SAVs. For instance, Fagnant & Kockelman [10] use a heuristic to move SAVs to a  
252 closer waiting traveler rather than the first waiting traveler. The dispatcher might also  
253 change the path of a traveling SAV to handle the demand.
- 254 3. If a SAV  $v \in V$  is inbound to  $d$ 's location, the dispatcher might assign  $v$  to service  $d$  if  
255 possible. However, the dispatcher should consider  $v$ 's estimated time of arrival (ETA).  
256 If  $v$ 's ETA results in unacceptable waiting time for  $d$ , the dispatcher may also send an  
257 empty SAV to  $d$  to reduce waiting time.

258 Regardless of the conditions chosen for each action, the dispatcher must ensure that the  
259 demand will be handled.

### 260 3.2.2 A SAV arrives at a centroid

261 When a SAV  $v \in V$  arrives at a centroid  $i \in Z$ , it has finished its assigned trip. This should  
262 result in two types of actions. First, if  $v$  is carrying any travelers destined for  $i$ , they should  
263 exit  $v$ . Second, the dispatcher should assign  $v$  to park at  $i$  or depart on another trip. There  
264 are several possibilities for this assignment:

- 265 1. If  $v$  still has passengers, it should continue to the next destination. If ride sharing is  
266 allowed and the capacity of  $v$  permits it, other passengers at  $i$  may wish to take  $v$  to  
267 reduce their waiting time.
- 268 2. If  $v$  is empty, and a traveler  $d \in D$  is waiting at  $i$  for a SAV, it is reasonable to assign  $v$   
269 to accept  $d$ .  $v$  may then proceed directly to  $d$ 's destination or, if dynamic ride-sharing  
270 is allowed, to another centroid to pick up another passenger.
- 271 3. If no travelers are waiting at  $i$  and  $v$  is empty, the dispatcher might assign  $v$  to pick  
272 up a traveler at a different centroid.

- 273 4. The dispatcher could also assign  $v$  to wait at  $i$  until needed for future demand, contin-  
274 gent on parking availability.
- 275 5. Finally, the dispatcher might assign  $v$  to preemptively relocate to handle predicted  
276 demand.

277 The conditions given above are reasonable but may not be necessary. Optimizing the as-  
278 signment of actions for the existing and predicted demand could use the possible actions  
279 in different ways. For example,  $v$  might be assigned to park at  $i$  to wait for the expected  
280 demand even if  $v$  is already carrying passengers. This optimization problem is similar to the  
281 class of vehicle routing problems, which are NP-hard. Therefore, solving this optimization  
282 is outside the scope of this paper, but later sections will present a heuristic.

### 283 3.3 Traffic flow simulator

284 The traffic flow simulator takes as input SAV trips and their departure times and determines  
285 the arrival times of SAVs at centroids. The primary output of the simulator is to trigger the  
286 event that an SAV arrived at a centroid at the appropriate time.

287 Because the SAV framework is built on the events of a traveler calling a SAV, and a SAV  
288 arriving at a centroid, the framework admits many flow propagation models. The major  
289 requirement is that the model be integrated into simulation. After departing, a SAV travels  
290 along its assigned path until reaching the destination centroid, at which point it triggers the  
291 arrival event. Therefore, the framework must track the SAV travel times to determine arrival  
292 times, but its travel time may be evaluated by a variety of flow models. For instance, the  
293 travel time could be set as a constant or through link performance functions. SAV movement  
294 may also be modeled through micro- or meso-simulation. Any stochasticity in the traffic flow  
295 model is compatible with this framework because the SAV triggers the event only after it  
296 arrives at its destination. Note that this framework is compatible with other vehicles on  
297 the road affecting congestion through link performance functions or simulation-based flow  
298 propagation.

299 Therefore, this SAV framework can be implemented with existing traffic models by mod-  
300 ifying them to trigger demand and centroid arrival events. To demonstrate this flexibility,  
301 the case study implements this framework on the simulation-based DTA model of Levin &  
302 Boyles [19].

## 303 4 Case study: framework implementation

304 This section describes the implementation of the SAV framework on a cell transmission  
305 model-based traffic simulator. Although Section 3 discussed how to implement SAVs in  
306 existing traffic simulators, the responses of the dispatcher to events were not specified for  
307 generality. The purpose of this section is to describe the specific traffic flow simulator and  
308 dispatcher logic used in our case study, including the heuristics for dynamic ride-sharing.  
309 Results using this implementation are presented in Section 5.

310 This case study assumes that all vehicles are SAVs: travelers do not have personal vehicles  
311 available. This was chosen to study the feasibility of switching to an entirely SAV-based travel  
312 model. Furthermore, a mix of SAVs and personal vehicles would complicate the route choice.  
313 Finding routes for personal vehicles would require solving DTA, and the many simulations  
314 needed to solve DTA would add computation time and complexity to the theoretical model.

## 315 4.1 Demand

316 This case study used personal vehicle trip tables from the morning peak to determine SAV  
317 traveler demand. Each vehicle trip was converted into a single traveler trip with the same  
318 origin, destination, and departure time. Although some of these vehicle trips may encompass  
319 multiple person trips, that information was not available. Furthermore, multiple persons us-  
320 ing the same vehicle would likely use the same SAV. Therefore, it would only affect situations  
321 in which SAV capacity was a limitation, such as dynamic ride-sharing.

322 For each trip, the demand module creates a traveler at the appropriate time. Although  
323 the demand is fixed, the SAV dispatcher is not programmed to take advantage of demand  
324 information. The dispatcher only responds to demand when a traveler was created.

325 In reality, travelers have more choices available. They could request a SAV in advance,  
326 specify time windows for departure or arrival, or change their departure time in response to  
327 expected travel times.

## 328 4.2 Traffic flow simulator

329 The traffic flow simulator uses the cell transmission model (CTM) [5,6], which is a Godunov  
330 approximation [14] to the hydrodynamic theory of traffic flow [23,24]. CTM discretizes links  
331 into *cells* of length  $u^f \Delta t$ , where  $u^f$  is the free flow speed and  $\Delta t$  is the simulation time step.  
332 Thus, vehicles can traverse at most one cell per time step. Congestion waves from bottlenecks  
333 or intersections travel backwards through the cells and reduce vehicle speeds. Since AVs  
334 increase capacity [16,26], this simulator use the CTM and flow-density relationship developed  
335 by Levin & Boyles [19]. Because all vehicles are SAVs, intersections were controlled using the  
336 reservation-based protocol of Dresner & Stone [7,8] for AVs. For computational tractability,  
337 the simulator used the conflict region node model of reservation-based intersection control  
338 proposed by Levin & Boyles [18].

339 CTM has been used in, and allows direct comparisons with, large-scale mesoscopic DTA  
340 simulators [29]. DTA models [4] typically assume that route choice is based on driver experi-  
341 ence. Each vehicle individually seeks its shortest route, resulting in a DUE. DTA algorithms  
342 typically consist of three steps, performed iteratively, to find a DUE assignment [20]. First,  
343 shortest paths are found for all origin-destination pairs. Then, a fraction of demand is  
344 assigned to the new shortest paths. Finally, travel times under the new assignment are  
345 evaluated through a mesoscopic flow model such as CTM.

346 Although DUE is based on the analytical static traffic assignment models, it requires  
347 further study to be formulated for SAV behavior due to stochasticity in the SAV trip table.  
348 We assume that the SAV dispatcher does not know travel demand or SAV travel times

349 perfectly. Therefore, the list of free SAVs at any given time is stochastic, which results in  
350 uncertainty in which SAV will be used to service new demand.

351 Therefore, we use a dynamic network loading (DNL) -based route assignment. Let  $\pi_{rs}$   
352 be the path stored by the dispatcher for travel from  $r$  to  $s$ . When a SAV departs to travel  
353 from  $r$  to  $s$ , it is assigned to the stored path  $\pi_{rs}$ . During simulation, when  $t \equiv 0 \pmod{\Delta\mathcal{T}}$ ,  
354 where  $\Delta\mathcal{T}$  is the update interval,  $\pi_{rs}$  is updated to be the shortest path from  $r$  to  $s$  based  
355 on average link travel times over the interval  $[t - \Delta\mathcal{T}, t)$ . Our experiments use  $\Delta\mathcal{T} = 1$   
356 minute. Note that the path update interval ( $\Delta\mathcal{T} = 1$  minute) is different from the traffic  
357 flow simulation time step ( $\Delta t = 6$  seconds).

### 358 4.3 SAV dispatcher

359 This section describes the specific logic used to assign SAVs in our case study. Although this  
360 is only a heuristic for the vehicle routing problem of servicing all travelers, vehicle routing  
361 problems in general are NP-hard and solving them in real time is unrealistic. Instead, we  
362 describe reasonable behaviors that SAVs could choose.

#### 363 4.3.1 A traveler calls a SAV

364 When a traveler  $d \in D$  calls a SAV at centroid  $i \in Z$ , the dispatcher first checks whether  
365 there are any SAVs already enroute to  $i$ . If a SAV enroute to  $i$  is free, or will drop off its  
366 last passenger at  $i$ , and its ETA at  $i$  is less than 10 minutes away, that SAV is assigned  
367 to service  $d$ . This is to reduce congestion resulting from sending more SAVs. (As Section  
368 5 will demonstrate, moving SAVs more frequently can result in a net travel time increase  
369 while decreasing waiting times due to congestion.) If there are multiple travelers waiting at  
370  $i$ , travelers are serviced in a first-come-first-serve (FCFS) order — with some exceptions for  
371 dynamic ride-sharing. Therefore, we look at the ETA of the SAV that would be assigned to  
372  $d$ , if one exists.

373 Otherwise, we search for the parked SAV that is closest (in travel time) to  $i$ . If it could  
374 arrive sooner than the ETA of the appropriate enroute SAV, it is assigned to travel to  $i$   
375 to provide service to  $d$ . This is a FCFS policy: the traveler that requests a SAV first will  
376 be the first to get picked up, even if the SAV could sooner reach a traveler departing later.  
377 Although Fagnant & Kockelman [10] initially restricted SAV assignments to those within  
378 5 minutes of travel to improve the system efficiency, FCFS is also a reasonable policy for  
379 dispatching SAVs. If all SAVs are busy, then  $d$  is added to the list of waiting travelers  $\mathcal{W}$ .

#### 380 4.3.2 A SAV arrives at a centroid

381 If a SAV  $v \in V$  is free after reaching centroid  $i \in Z$  (either because  $v$  is empty, or because  $v$   
382 drops off all passengers at  $i$ ), and there are waiting travelers at  $i$ , then it is assigned to carry  
383 the longest waiting traveler. Note that  $v$  may not be the same SAV that was dispatched  
384 to that traveler. Due to stochasticity in the flow propagation model, it is possible that the  
385 order of arrival of SAVs may differ. However, there is no significant difference between two

386 free SAVs in terms of carrying a single traveler. Therefore, we assign them to travelers in  
387 FCFS order.

388 If  $v$  still has passengers after reaching  $i$  (which is possible when dynamic ride-sharing  
389 is permitted), then  $v$  is assigned to travel to the next passenger’s destination. However,  
390 travelers waiting at  $i$  have the option of entering  $v$  if it makes sense for their destination.  
391 This is discussed further in Section 4.4.

392 If  $v$  is free after reaching  $i$  and no demand is waiting at  $i$ , then  $v$  is dispatched to the  
393 longest-waiting traveler in  $\mathcal{W}$ . If multiple SAVs become free at the same time, the one closest  
394 to the longest-waiting traveler in  $\mathcal{W}$  will be sent. If  $\mathcal{W}$  is empty, then  $v$  will park at  $i$  until  
395 needed. We assume for this study that centroids have infinite parking space, as there are no  
396 personal vehicles in this network. However, it would be possible to model limited parking  
397 by assigning  $v$  to travel somewhere else if parking was not available at  $i$ .

## 398 4.4 Dynamic ride-sharing

399 We also consider the possibility of dynamic ride-sharing. Following the principle of FCFS, we  
400 give precedence to the longest-waiting traveler. However, we allow other passengers to enter  
401 the SAV if they are traveling to the same, or a close destination. Specifically, suppose that  
402 the SAV  $v \in V$  is initially empty, and the longest-waiting traveler at  $i \in Z$  is  $d_0$ , traveling  
403 from  $i$  to  $j \in Z$ . If there is another traveler  $d_1$  also traveling from  $i$  to  $j$ , then  $d_1$  may take  
404 the same SAV. If there is a traveler  $d_2$  traveling from  $i$  to  $k \in Z$ , and there is room in the  
405 SAV,  $d_2$  may also take the same SAV if the additional travel time is sufficiently low. Let  $t_{ij}$   
406 be the expected travel time from  $i$  to  $j$ . Then  $d_2$  will take the SAV if  $t_{ij} + t_{jk} \leq (1 + \epsilon)t_{ik}$ .  
407 Otherwise,  $d_2$  will wait at  $i$ . If  $d_2$  decides to take the SAV, then any other waiting travelers  
408 at  $i$  also traveling from  $i$  to  $k$  may enter the SAV. Although this violates FCFS, this is  
409 permitted because it does not impose any additional travel time on the SAV.

410 This offer is extended, in FCFS order, for all travelers waiting at  $i$  until  $v$  is full. For  
411 instance, suppose a passenger  $d_3$  departing after  $d_2$  is traveling from  $i$  to  $l \in Z$ . Because of  
412 FCFS,  $v$  must service  $d_2$  first, but if  $t_{ij} + t_{jk} + t_{kl} \leq (1 + \epsilon)t_{il}$ , then  $d_3$  will still take SAV  $v$   
413 from  $i$ .

414 The logic is slightly different when  $v$  arrives at  $i$  already carrying a passenger. In that  
415 case, precedence is given to all passengers already in  $v$  because they have been traveling.  
416 However, travelers in  $i$  may enter  $v$  — at the back of the queue — if the additional travel  
417 time is less than  $\epsilon$  of the direct travel time.

418 The problem of dynamic ride-sharing is a vehicle routing problem with all SAVs. In  
419 general, vehicle routing problems can admit solutions in which a SAV picks up several pas-  
420 sengers before dropping any off. The heuristic in this case study does not do that due to  
421 complexity, although that behavior could certainly be implemented within this framework.  
422 In practice, due to the necessity of tractability when solving vehicle routing problems in  
423 real-time in response to demand, similar simple heuristics are likely to be used. Even with  
424 this restricted form of dynamic ride-sharing, the benefits over non-ride-sharing SAVs are  
425 significant, as shown in Section 5.

## 5 Case study: experimental results

We performed several sets of experiments to study how SAVs (Sections 5.2 through 5.3) perform relative to personal vehicles (Section 5.1), and how the dynamic ride-sharing heuristic affects performance. Our experiments were performed on the downtown Austin network, shown in Figure 4. It consists of a downtown grid with freeway and arterial corridors. It has 171 zones, 546 intersections, 1,247 links, and 62,836 trips over 2 hours in the AM peak. The centroids are significantly disaggregated for this downtown region, so we did not include intra-zonal trips in the trip table. The network was calibrated by the Network Modeling Center to match traffic data from the Capital Area Metropolitan Planning Organization.

This is only a subnetwork of the larger Austin region, which has 1.2 million trips. This subnetwork was used because computation times were around 30–40 seconds per scenario on an Intel Xeon running at 3.33 GHz (implemented in Java), allowing many scenarios to be studied. However, many trips bound for the downtown grid originate from outside the subnetwork region. We approximated them as arriving from one of the subnetwork boundaries. The 62,836 trips within the downtown subnetwork is sufficient for a large-scale, realistic study of SAVs.

Initially, SAVs were distributed proportionally to productions: centroid  $i \in Z$  started with  $|V| \frac{P_i}{\sum_{i' \in Z} P_{i'}}$  parked SAVs, which corresponds to  $\Delta V_i = 0$ . We assumed that all SAVs could be relocated overnight to fulfill these proportions at the start of the AM peak. (Preemptive relocation is a strategy for relocating SAVs *during* the AM peak — while travelers are requesting SAVs.)

Fagnant & Kockelman [10] used a seeding run to determine the minimum number of SAVs necessary to service all travelers. However, a seeding run may have biased the number of SAVs to be lower. Instead of a seeding run, we performed sensitivity analyses to study how increasing numbers of SAVs affected level of service. In some scenarios (such as dynamic ride-sharing) we observed that fewer numbers of SAVs performed better due to lower congestion. In other scenarios, greater numbers of SAVs improved service. The following charts contain experiments using between 1000 and 60,000 SAVs, with increments of 500. For some scenarios, the range was reduced to numbers of SAVs that could provide service to all travelers within 6 hours because service was limited by having too few SAVs or too much congestion.

### 5.1 Personal vehicles

For comparison, we also considered two personal vehicle scenarios on the downtown Austin network:

1. All travelers drive personal non-autonomous vehicles. This represents current traffic conditions, and shows
2. All travelers use personal AVs, and use AV capacity and intersection improvements. This is an alternative to SAVs in which travelers own the AVs.

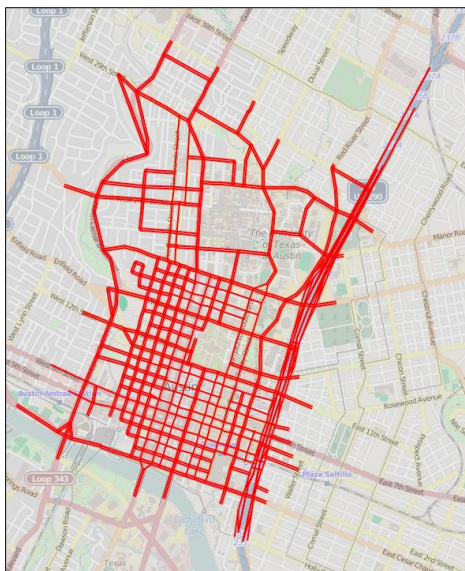


Figure 4: Downtown Austin network

Table 1: Results from personal vehicle scenarios

Scenario	Avg. travel time	Vehicle miles traveled
Personal conventional vehicles	15.24 min	146096 mi
Personal autonomous vehicles	4.12 min	142455 mi

464 For the private vehicle scenarios, we assumed that travelers chose routes to minimize their  
 465 own travel time, resulting in a DUE. Therefore, we used DTA to find route choice for personal  
 466 vehicle scenarios.

467 One potential issue with comparing these personal vehicle scenarios with SAVs is the  
 468 different methods used for route choice. For personal vehicles, we assumed DUE behavior,  
 469 and for SAVs, we assumed DNL behavior determined by the SAV dispatcher. DUE is widely  
 470 accepted for modeling personal vehicle behavior [4]. DNL was used for SAVs because the  
 471 SAV dispatcher is modeled to react to travel demand as it appears. Therefore, to handle  
 472 stochastic demand, the SAV dispatcher should rely on current rather than historical traffic  
 473 conditions in its route assignments. (Furthermore, a traffic assignment problem has not been  
 474 formulated for SAVs, and consequently it is not known how to solve DTA for SAVs.)

475 Results from personal vehicle scenarios are shown in Table 1. Overall, when using per-  
 476 sonal vehicles with traffic signals, travelers experienced an average travel time of 15.24 min-  
 477 utes. When signals were replaced with reservation controls, average travel times were reduced  
 478 to 4.12 minutes. Since the adoption of reservation controls may be difficult or inefficient if  
 479 a significant proportion of personal vehicles are not autonomous, both personal vehicle sce-  
 480 narios may be reasonable for comparison against SAVs. We assume that if SAVs were to  
 481 replace all personal vehicles, reservation controls would be used.

## 5.2 Shared autonomous vehicles

The initial SAV scenario did not include dynamic ride-sharing. Figure 5 shows travel time results with 17,500 to 60,000 total SAVs available. Fewer numbers of SAVs were found to be insufficient to service the 2 hours of travel demand after 6 hours. Greater numbers of SAVs reduced both waiting time and in-vehicle travel time. With more SAVs, more vehicles were available near traveler origins, and fewer empty repositioning trips reduced congestion.

As the number of SAVs increased, waiting time decreased consistently, although with diminishing returns. With 39,500 or more SAVs, average waiting times were below 1 minute. Waiting times approached 0 because SAVs were assumed to be initially distributed according to trip productions. Therefore, with 62,836 or more SAVs, waiting times would be 0. Of course, one of the goals of SAVs is to reduce the total number of vehicles in [10].

Because the demand is from the AM peak, much of the waiting time results from SAVs carrying travelers to the downtown region then making an empty repositioning trip to the next traveler’s origin. However, waiting times were only 10.3 minutes with 17,500 SAVs. With 25,500 or more SAVs, average waiting times were less than 5 minutes. These average waiting times could be acceptable to travelers.

The average in-vehicle travel time (IVTT) was higher than the personal vehicle scenarios at low numbers of SAVs. This shows that a small SAV fleet requires many empty repositioning trips to service travelers. The empty repositioning trips result in greater demand and therefore congestion. This is particularly relevant for peak hour scenarios, which result in the greatest number of empty repositioning trips because most trips are to or from the central business district. SAV models that do not include realistic travel time predictions would not be able to predict the congestion caused by a small SAV fleet.

This AM peak hour scenario required far more SAVs than 1 per 9.3 travelers [12]. 1 SAV could replace at most 3.6 personal vehicles, and total travel time was significantly higher there. SAV fleet size is likely to be determined by peak hour demand because peak hour travel patterns are the most difficult to serve with SAVs.

However, with only 22,000 SAVs, the average IVTT was less than the personal non-AV scenario of 15.24 minutes (Table 1). The average IVTT never decreased below 9.8 minutes — higher than the 4.12 minutes of the personal AV scenario, but small enough to be feasible for travelers. This was probably due to the route choice heuristic used in this scenario. Personal AVs used DUE behavior, whereas SAVs did not. Better heuristics for SAV routing could therefore decrease the IVTT further for SAVs. Still, the average IVTT was not substantially higher than the personal AV scenario.

Vehicle miles traveled (VMT) and empty VMT — miles traveled while not carrying any passengers — decreased at the same rate as the number of SAVs increased (Figure 5). This indicates that the difference was primarily due to less repositioning trips to pick up the next traveler, rather than changes in route choice. It is intuitive that as the number of SAVs increased, the average distance between a waiting traveler and the nearest (in travel time) available SAV would decrease. The average passenger miles traveled was consistently 2.27 miles.



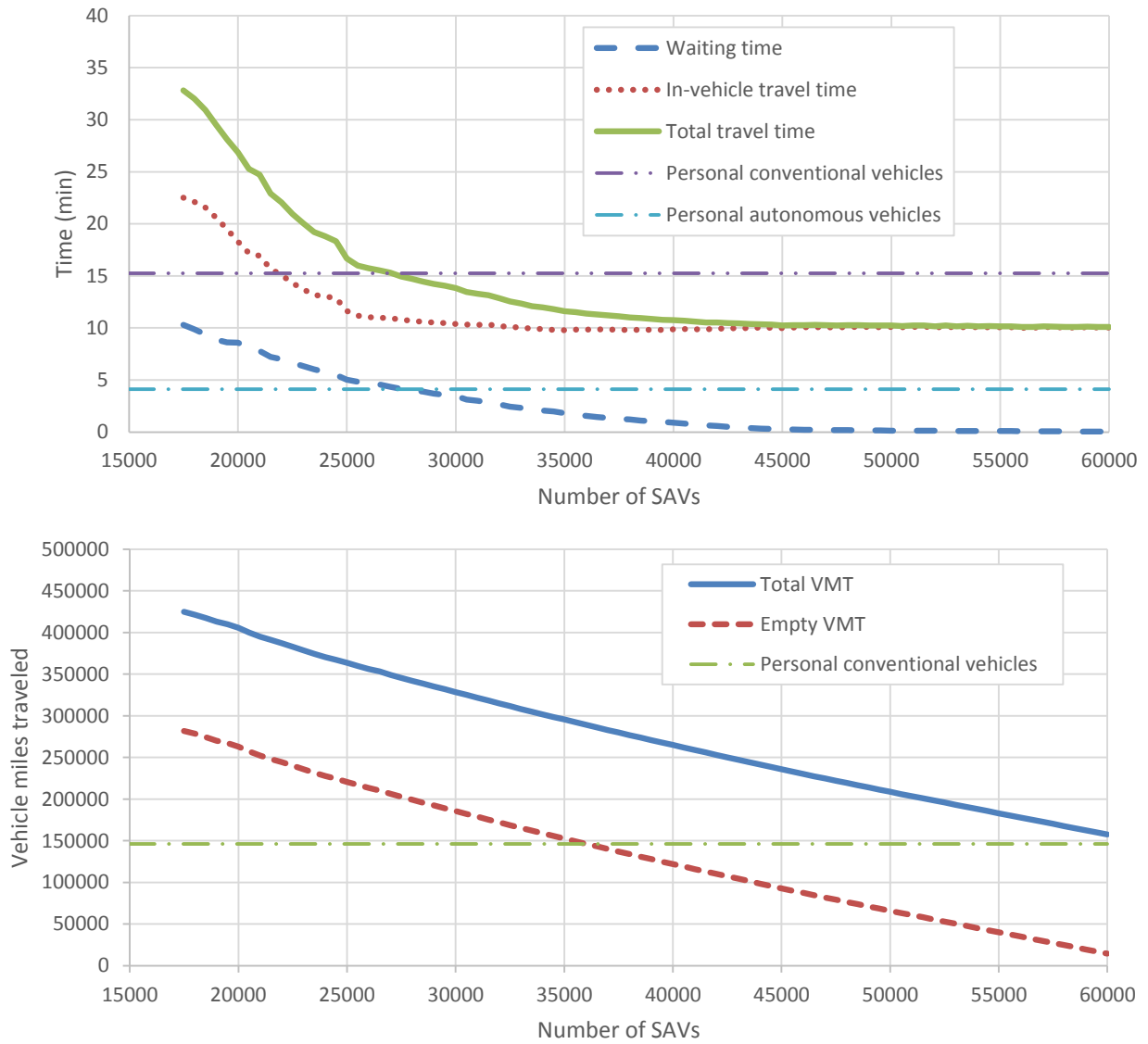


Figure 5: Travel time and VMT for the base SAV scenario

### 5.3 Dynamic ride-sharing

Dynamic ride-sharing greatly affected level of service for travelers as shown in Figure 6. With dynamic ride-sharing, 1000 SAVs were actually sufficient to service all demand. Each SAV could carry up to 4 passengers, although they would travel with less if no travelers were waiting. However, because most trips were to the central business district, SAVs could easily combine trips because traveler destinations were relatively close. Surprisingly, optimal service was provided with just 2000 SAVs, or a ratio of 1 SAV to 31.4 travelers. This is significantly higher than the 1 SAV to 9.3 travelers [12] although of course here each SAV was probably carrying 3 to 4 passengers.

The least average total travel time was 6.46 minutes with 2000 SAVs, comparable with the 4.12 minutes with the personal AV scenario (Table 1). 5.41 minutes was due to IVTT, with 1.04 minutes due to waiting time. These travel and waiting times might be further reduced with a better heuristic for dynamic ride-sharing. Therefore, with such a low travel time, SAVs with dynamic ride-sharing could be an effective replacement for personal AVs. Furthermore, the size of the SAV fleet used is so small relative to the number of travelers that full replacement might be feasible. The cost per traveler are also likely to be significantly reduced due to car-sharing and the lack of driver. Further study in different demand scenarios and on different networks is needed, but this result suggests that SAVs could be a cost-effective form of paratransit with a high level of service.

Waiting times were consistently low with 2000 or more SAVs. This is probably because most travelers had relatively close destinations, so ride-sharing was frequently used. Strangely, IVTT peaked at 17.54 minutes with 11,000 SAVs. This was likely because SAVs did not wait around for ride-sharing with later-departing travelers. Therefore, the 11,000 SAVs made more trips, carrying fewer travelers per trip, and increased congestion. Figure 7 shows that passenger miles traveled increased as the number of SAVs increased because ride-sharing was used less. With greater than 11,000 SAVs, travel times decreased because less empty repositioning trips were needed, decreasing vehicle demand. VMT, and empty repositioning miles traveled, was highest around 14,500 SAVs (Figure 6). With our heuristic, a fleet of between 5500 and 17,500 SAVs was less efficient than a smaller fleet. Therefore, future work on SAVs should study more effective heuristics for the dynamic ride-sharing problem.

## 6 Conclusions

This paper presented an event-based framework for implementing SAV behavior in existing traffic simulation models. The framework relies on two events: travelers calling SAVs, and SAVs arriving at centroids, that are orthogonal to traffic flow models. This allows comparisons with personal vehicle scenarios through solving traffic assignment in the same simulator. We implemented this SAV framework on a cell transmission model-based dynamic traffic assignment simulator as well as a heuristic approach to dynamic ride-sharing. Then, we studied replacing personal vehicles with SAVs in the downtown Austin network with AM

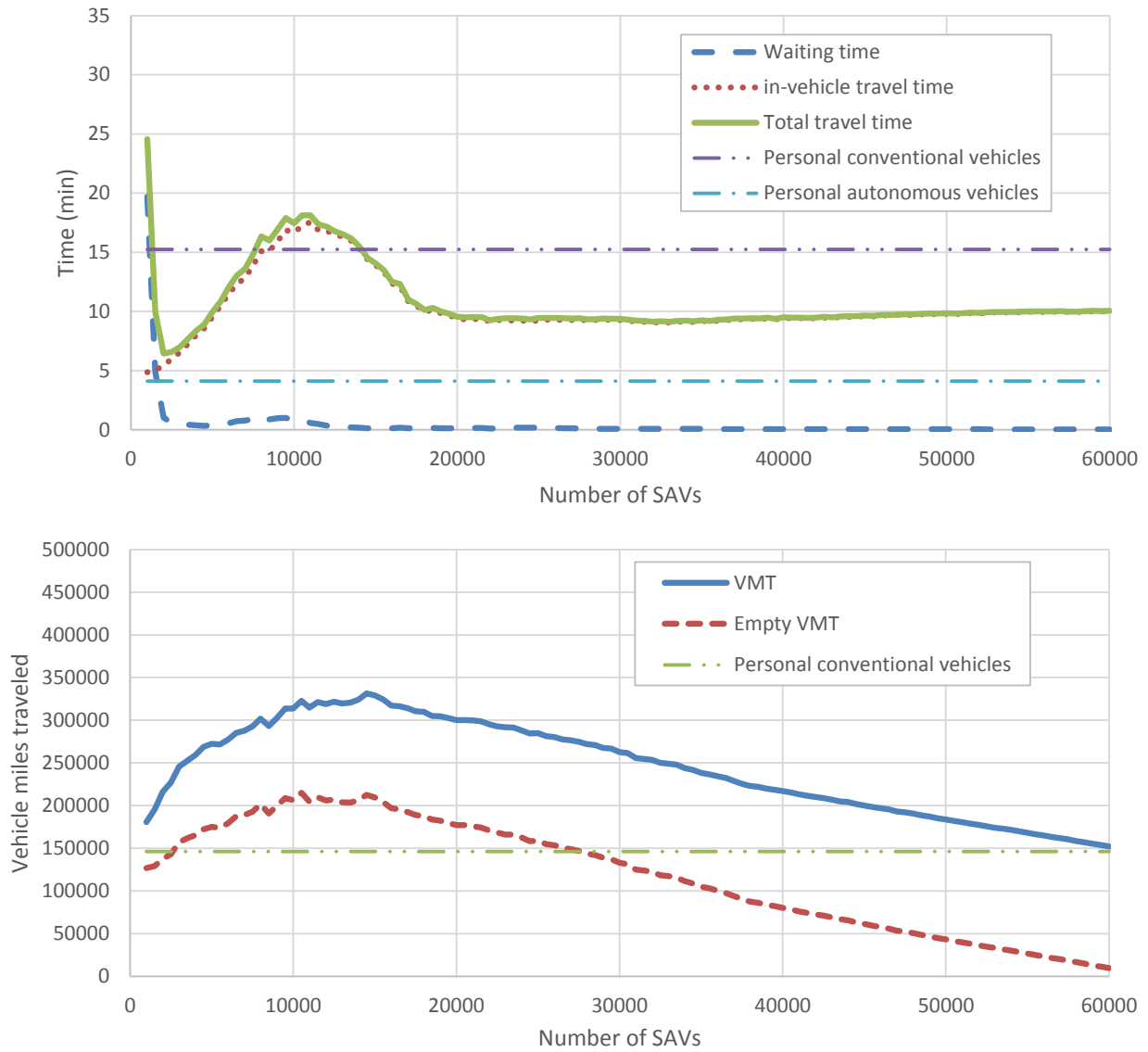


Figure 6: Travel time and VMT for the dynamic ride-sharing scenario

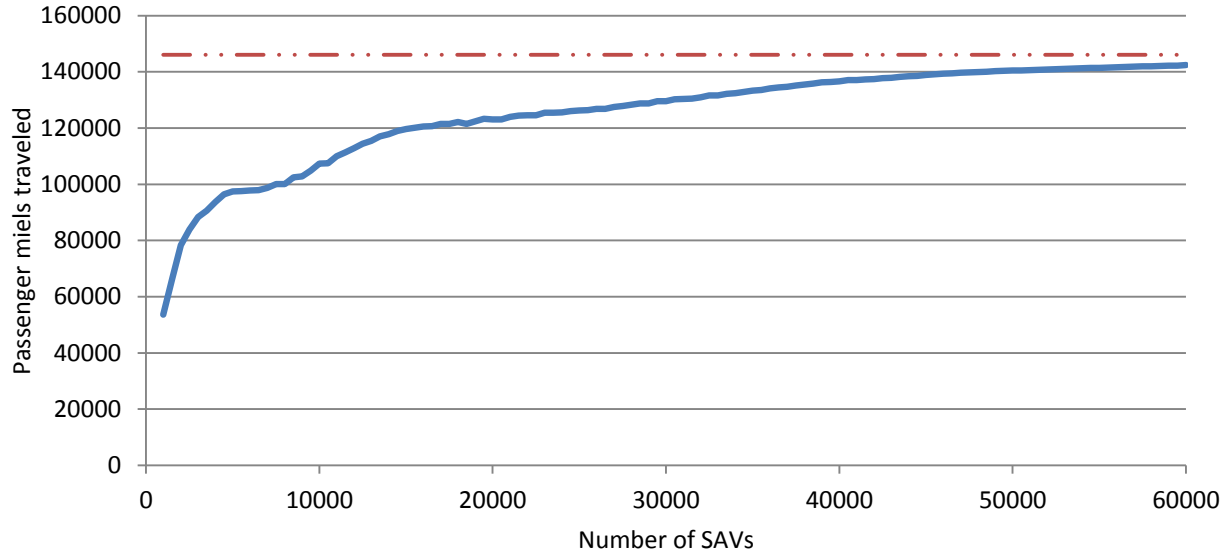


Figure 7: Passenger miles traveled for the dynamic ride-sharing scenario

562 peak demand. Most SAV scenarios resulted in greater congestion due to empty repositioning  
 563 trips to reach travelers' origins.

564 Using SAVs without dynamic ride-sharing resulted in higher travel time than personal  
 565 AVs. These levels of service appear to be lower than predicted by previous studies. Further-  
 566 more, a much larger SAV fleet size was needed for the AM peak. Although this paper used  
 567 heuristics to solve the vehicle routing problem, finding an optimal solution in real-time in  
 568 response to demand is impractical because the vehicle routing problem is NP-hard. Further-  
 569 more, previous studies also used similar heuristics. Therefore, these results demonstrate the  
 570 importance of using realistic traffic flow models to study the additional congestion resulting  
 571 from SAVs, and comparing SAVs with personal vehicles with a common traffic flow model.  
 572 This paper also provides the framework to integrate SAV behavior into such models.

573 However, dynamic ride-sharing was highly effective at reducing congestion by combining  
 574 traveler trips. Interestingly, ride-sharing had the best travel times when the number of  
 575 SAVs was small (2000 SAVs providing service to 62,836 travelers), and these travel times  
 576 were comparable or improved over personal vehicle scenarios. This shows that with effective  
 577 routing heuristics and the right fleet size, SAVs could replace personal vehicles as paratransit  
 578 or individual taxis.

579 Future studies should analyze how SAVs perform in a greater variety of scenarios, in-  
 580 cluding varying demand and network topology. The experiments in this paper focused on a  
 581 downtown grid network; a more suburban area with greater distance trips may be affected  
 582 differently. This framework could also be used to study replacing traditional taxi service with  
 583 SAVs. Taxis are typically constantly moving, which might increase congestion but decrease  
 584 wait times. Additionally, better methods for vehicle routing and dynamic ride-sharing could  
 585 improve SAV service, although any solution algorithms will have to be tractable for real-time

586 execution in response to stochastic demand.

587 In addition, models using this framework could be used for travel demand and mode  
588 choice analyses. Travelers' trip choices typically depend on travel times, which could be  
589 greatly increased from congestion caused by SAVs. Many previous studies have assumed  
590 that all personal vehicle travel is replaced by SAVs [10–12]. In reality, SAVs add another  
591 mode option to personal vehicles and mass transit, and a fraction of travelers will choose  
592 each mode. The utility for each mode depends on travel times, for which congestion is a  
593 major factor. In particular, SAV congestion and routing affects both in-vehicle travel times  
594 as well as time spent waiting for pickup. With SAVs comprising a large fraction of vehicles  
595 on the road, SAVs will also affect the travel times of other modes as well. Of course, the  
596 number of travelers choosing the SAV mode will correspondingly affect the congestion caused  
597 by SAVs. To find mutually consistent travel demand, mode choice, and traffic congestion  
598 solutions, a SAV model with realistic congestion should be integrated into planning models  
599 to better predict the impacts of SAVs on city traffic patterns.

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